Springer

KSME

Journal of Mechanical Science and Technology 25 (5) (2011) 1071~1079
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-011-0223-4

A new method based on LPP and NSGA-II for multiobjective robust collaborative optimization †

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(Manuscript Received March 24, 2010; Revised December 19, 2010; Accepted January 16, 2011)

Abstract

The multiobjective robust collaborative optimization framework consists of optimization both at the system and autonomous subsystem levels. Linear physical programming is used in the system level optimization, which avoids the difficulty in choosing the multidimensional Pareto set. The non-dominated sorting genetic algorithm (NSGA-II) is used in the subsystem optimization with physical objectives. The interdisciplinary incompatibility function and physical objectives have different priority levels. At the first priority level, the best individual should be in the feasible region of the subsystem. At the second priority level, the interdisciplinary incompatibility function of the best individual should be no more than the feasibility threshold. The physical objectives are improved after the achievement of the above levels. A method for producing initial population with feasibility and diversity is proposed to improve the calculation efficiency and accuracy of the subsystem optimization at the first priority level. A method for setting dynamic feasibility threshold is proposed for the non-dominated sorting to help the physical objectives to obtain better solutions at the second priority level. Finally, the results of the speed reducer show that the presented method is efficient.

Keywords: Multidisciplinary design optimization; Collaborative optimization; Multiobjective; Linear physical programming; NSGA-II

1. Introduction

Multidisciplinary design optimization (MDO) is a concurrent engineering design tool for large-scale, complex system design that has recently attracted a great deal of attention. Collaborative optimization (CO) is an approach to MDO problems. The key concept in the CO approach is the decomposition of the design problem into two levels, the system level and the subsystem level. The system level optimizer is used to minimize the system level objective while satisfying consistency requirements among the disciplines by enforcing equality constraints at the system level that coordinate the interdisciplinary couplings [1]. However, there often exist uncontrollable uncertainties in parameters of an MDO problem [2]. Uncertainties may exist not only in each discipline but also in the couplings among disciplines and hence methods for handling uncertainties within and across disciplines have become quite important [3]. For probabilistic based approach, Du et al. [4] developed the system uncertainty analysis (SUA) and concurrent subsystem uncertainty analysis (CSSUA) methods. For interval analysis based approach, Gu et al. [5, 6] developed the implicit uncertainty propagation (IUP) method. In the robust MDO formation developed by Li et al. [3], the upper and lower bounds of interdisciplinary coupling variable variations were set by a decision maker (DM). Wang et al. [7] developed the generalized dynamic constraint network to analysis and manage uncertainties. The methods developed by Du and Gu have obtained the most attention.

More importantly, due to the multiobjective nature of some MDO problems, recent work has focused on formulating the MDO problem to resolve tradeoff between multiple, conflicting objectives. Tappeta et al. [8] used the weighted-sum method to resolve the multiobjective collaborative optimization (MOCO) problem. McAllister et al. [9-11] applied the goal programming and linear physical programming (LPP) approaches to resolve the MOCO problem. Huang et al. [12] used the fuzzy satisfaction degree and fuzzy sufficiency degree methods to handle the MOCO problem. Vikrant et al. [13] and Sebastien et al. [14] used the multiobjective evolutionary algorithm (MOEA) method at both the system and subsystem levels to handle the MOCO problem. In most previous studies on MOCO problems, the subsystem objective only aims at minimizing the interdisciplinary incompatibility function, and is not related to the physical problem [13]. How-

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ever, the study on MOCO problems with physical objectives in subsystems is important because it accounts for the case when one or more objectives are important and considered at a subsystem but not at the system level [3, 10].

Our work focuses on the multiobjective robust collaborative optimization (MORCO), which has multiple objectives both at the system and subsystem levels. To avoid the difficulty in choosing the multidimensional Pareto set, the LPP method is used in the system level optimization instead of the MOEA method. The NSGA-II method is used in the subsystem optimization with physical objectives. However, the interdisciplinary incompatibility function and physical objectives have different priority levels. To improve the calculation efficiency and accuracy, a method for producing initial population with feasibility and diversity is presented. To obtain better solutions for subsystem physical objectives, a method for setting dynamic feasibility threshold is presented.

This paper is organized as follows. Section 2 gives the MORCO formation. Section 3 describes the system level optimization based on LPP. Section 4 presents the subsystem optimization based on NSGA-II. Section 5 uses the speed reducer example to demonstrate the applicability of the proposed method. Finally, section 6 gives the concluding remarks.

2. Multiobjective robust collaborative optimization formations

In the robust collaborative optimization (RCO) framework, the uncertainty range of design variable x is assumed as Δx. The model error is represented by $\Delta\delta$, which is assumed to be proportional to the model output. The variant value Δy of state variable y should be estimated within the bi-level optimization framework. In the IUP method [5, 6], Δy is estimated through Eq. (1).

$$\begin{Bmatrix}\Delta y_1\\ \Delta y_2\\ \Delta y_3\end{Bmatrix}=\begin{Bmatrix}\left|\frac{dy_1}{dx}\right|\\ \left|\frac{dy_2}{dx}\right|\\ \left|\frac{dy_3}{dx}\right|\end{Bmatrix}|\Delta x|+\begin{bmatrix}|B_{11}| & |B_{12}| & |B_{13}|\\ |B_{21}| & |B_{22}| & |B_{23}|\\ |B_{31}| & |B_{32}| & |B_{33}|\end{bmatrix}\cdot\begin{Bmatrix}|\delta_1(x,y_2,y_3)|\\ |\delta_2(x,y_1,y_3)|\\ |\delta_3(x,y_1,y_2)|\end{Bmatrix} \quad (1)$$

where $y_1 \sim y_3$ are the state outputs of discipline design tools $T_1 \sim T_3$. $B_{11} \sim B_{33}$ are estimated by the local partial derivatives of $T_1 \sim T_3$ with respect to $y_1 \sim y_3$. $\delta_1 \sim \delta_3$ are the bias errors associated with $T_1 \sim T_3$. $\frac{dy_1}{dx} \sim \frac{dy_3}{dx}$ are approximated by global sensitivity equation detailed in Eq. (2).

$$\begin{Bmatrix}\frac{dy_1}{dx}\\ \frac{dy_2}{dx}\\ \frac{dy_3}{dx}\end{Bmatrix}=\begin{bmatrix}I_1 & -\frac{\partial T_1}{\partial y_2} & -\frac{\partial T_1}{\partial y_3}\\ -\frac{\partial T_2}{\partial y_1} & I_2 & -\frac{\partial T_2}{\partial y_3}\\ -\frac{\partial T_3}{\partial y_1} & -\frac{\partial T_3}{\partial y_2} & I_3\end{bmatrix}^{-1}\begin{Bmatrix}\frac{\partial T_1}{\partial x}\\ \frac{\partial T_2}{\partial x}\\ \frac{\partial T_3}{\partial x}\end{Bmatrix} \quad (2)$$

where $I_1 \sim I_3$ are the identity matrixes.

In the CO framework, the auxiliary design variable x_{aux} is introduced as the additional design variable to replace the state variable y. In this paper, x_{aux} is also introduced to replace the variant value Δy of state variable y to avoid the calculation of Eq. (1), (2). Then the MORCO formations are given as follows:

System level optimization:

$$\min f_{0p}(\mathbf{z}) \quad (3)$$

$$\min \Delta f_{0p}(\mathbf{z}) \quad (4)$$

$$\Delta f_{0p}(\mathbf{z})=\sum_{j=1}^{s_{sh}}\left|\frac{\partial f_{0p}(\mathbf{z})}{\partial z_j}\Delta z_j\right|+\sum_{j=s_{sh}+1}^{s_{sh}+s_{aux}}\left|\frac{\partial f_{0p}(\mathbf{z})}{\partial z_j}\Delta z_j\right|+\Delta\delta_0 \quad (5)$$

$$s.t.\ J_i^*(\mathbf{z})=\sum_{j=1}^{s_{ish}}(z_j-x_{ij}^*)^2+\sum_{j=s_{ish}+1}^{s_{ish}+s_{iaux}}(z_j-x_{ij}^*)^2+\sum_{j=s_{ish}+s_{iaux}+1}^{s_{ish}+2s_{iaux}}(z_j-x_{ij}^*)^2=0$$

$$p=1,2,\cdots,n_s, \quad i=1,2,\cdots,n,$$

where f_{0p} is the p th system level objective function. Δf_{0p} is the variant function of f_{0p}. n_s is the number of system level objective functions. n is the number of subsystems. $\mathbf{z}$ is the system level design vector. z_j is the j th design variable of $\mathbf{z}$. Δz_j is the variant value of z_j. x_{ij}^* is the optimization result of the j th design variable of subsystem i. s_{sh} is the number of system level sharing variables. s_{aux} is the number of system level auxiliary variables. s_{ish} is the number of sharing variables of subsystem i. s_{iaux} is the number of auxiliary design variables of subsystem i. $\Delta\delta_0$ is the model error.

Subsystem optimization:

$$min\ J_i(\mathbf{x_i})=\sum_{j=1}^{s_{ish}}(x_{ij}-z_j^*)^2+\sum_{j=s_{ish}+1}^{s_{ish}+s_{iaux}}(x_{ij}-z_j^*)^2+\sum_{j=s_{ish}+s_{iaux}+1}^{s_{ish}+2s_{iaux}}(x_{ij}-z_j^*)^2 \quad (6)$$

$$\min f_{iq}(\mathbf{x_i}) \quad (7)$$

$$\min \Delta f_{iq}(\mathbf{x_i}) \quad (8)$$

$$\Delta f_{iq}(\mathbf{x_i})=\sum_{j=1}^{s_{ish}}\left|\frac{\partial f_{iq}(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|+\sum_{j=s_{ish}+1}^{s_{ish}+s_{iaux}}\left|\frac{\partial f_{iq}(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|+\sum_{j=s_{ish}+s_{iaux}+1}^{s_{ish}+s_{iaux}+s_{ilocal}}\left|\frac{\partial f_{iq}(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right| \quad (9)$$

$$s.t.\ g_i(\mathbf{x_i})+\Delta g_i(\mathbf{x_i})\le 0$$

$$\Delta g_i(\mathbf{x_i})=\sum_{j=1}^{s_{ish}}\left|\frac{\partial g_i(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|+\sum_{j=s_{ish}+1}^{s_{ish}+s_{iaux}}\left|\frac{\partial g_i(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|+\sum_{j=s_{ish}+s_{iaux}+1}^{s_{ish}+s_{iaux}+s_{ilocal}}\left|\frac{\partial g_i(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|$$

$$x_{iiaux}=T_i(\mathbf{x}_{ish},\mathbf{x}_{iaux},\mathbf{x}_{ilocal})$$

$$x_{ii\Delta aux}=\sum_{j=1}^{s_{ish}}\left|\frac{\partial T_i(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|+\sum_{\substack{j=s_{ish}+1\\ j\neq iaux}}^{s_{ish}+s_{iaux}}\left|\frac{\partial T_i(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|+\sum_{j=s_{ish}+s_{iaux}+1}^{s_{ish}+s_{iaux}+s_{ilocal}}\left|\frac{\partial T_i(\mathbf{x_i})}{\partial x_{ij}}\Delta x_{ij}\right|+\Delta\delta_i$$
$$q=1,2,\cdots,n_t$$

where f_{iq} is the q th objective function of subsystem i. Δf_{iq} is the variant function of f_{iq}. $\mathbf{x_i}$ is the design vector of subsystem i. x_{ij} is the j th design variable of $\mathbf{x_i}$. Δx_{ij} is the variant value of x_{ij}. z_j^* is the j th target value allocated by the system level. T_i is the discipline design tool. x_{iiaux} is the output of T_i. $\mathbf{x}_{ish}$ is the sharing design vector of subsystem i. $\mathbf{x}_{iaux}$ is the auxiliary design vector of subsystem i. $\mathbf{x}_{ilocal}$ is the local design vector of subsystem i. s_{ilocal} is the number of local design variables. $x_{ii\Delta aux}$ is the variant value of x_{iiaux}. $\Delta\delta_i$ is the model error. n_t is the number of subsystem objective functions.

3. System level optimization based on LPP

3.1 LPP description

LPP is an engineering method to deal with multiobjective optimization problems by using the designer's preference [15]. With the LPP procedure, the designer expresses his/her preferences with respect to each criterion using four different classes: (i) Smaller-Is-Better(1S); (ii) Larger-Is-Better(2S); (iii) Value-Is-Better(3S); (iv) Range-Is-Better(4S). Fig. 1 presents the depiction. Z_p is the class function which is smaller-better to each class. f_p is the value of the criterion under consideration.

3.2 System level optimization

The system level robust optimization based on LPP is presented as follows:

$$\min F(\mathbf{z})=\sum_{p=1}^{2n_s}\sum_{s=2}^{5}(\tilde{w}_{ps}^{-}d_{ps}^{-}+\tilde{w}_{ps}^{+}d_{ps}^{+}), \tag{10}$$

$s.t.\ f_{0p}(\mathbf{z})-d_{ps}^{+}\le t_{p(s-1)}^{+}$, $f_{0p}(\mathbf{z})\le t_{p5}^{+}$, $d_{ps}^{+}\ge 0$
(Classes 1S, 3S, 4S),
$f_{0p}(\mathbf{z})+d_{ps}^{-}\ge t_{p(s-1)}^{-}$, $f_{0p}(\mathbf{z})\ge t_{p5}^{-}$, $d_{ps}^{-}\ge 0$
(Classes 2S, 3S, 4S)

where d_{ps}^{-} denotes the negative deviation value between f_{0p} and $t_{p(s-1)}^{-}$, and d_{ps}^{+} denotes the positive deviation value between f_{0p} and $t_{p(s-1)}^{+}$. The calculation process of the weight $\tilde{w}_{ps}^{-}$ and $\tilde{w}_{ps}^{+}$ is given in Ref. [11]. The main process is as follows:

The change in Z_p across the s th range is given by

$$\tilde{Z}_s=Z_s-Z_{s-1},\quad 2\le s\le 5. \tag{11}$$

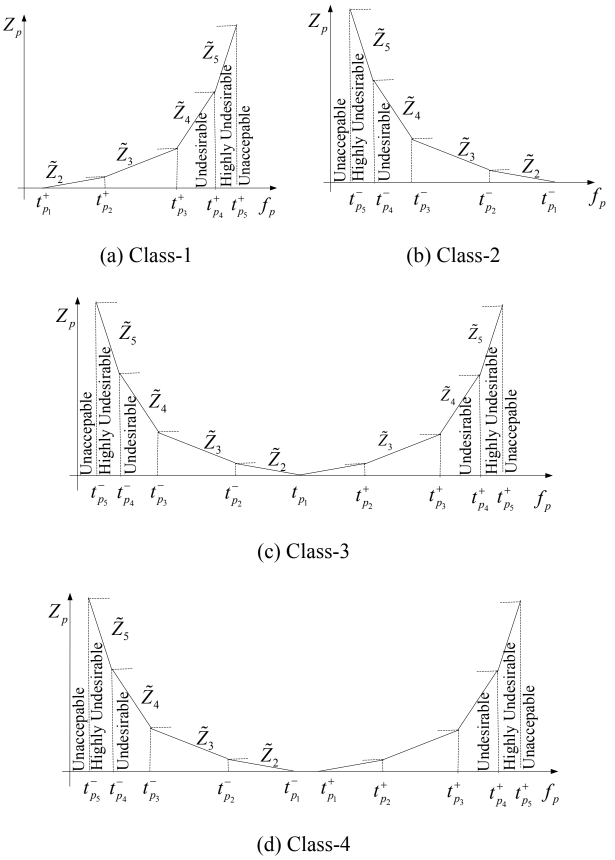


Fig. 1. Linear physical programming class function regions.

To enforce the OVO rule, the relationship is given by

$$\tilde{Z}_s=\theta(2n_s-1)\tilde{Z}_{s-1},\quad 2\le s\le 5,\quad \theta>1, \tag{12}$$

where θ is the convexity parameter. The length of the s th range of the p th objective is defined by

$$\tilde{t}_{ps}^{+}=t_{ps}^{+}-t_{p(s-1)}^{+},\quad \tilde{t}_{ps}^{-}=t_{ps}^{-}-t_{p(s-1)}^{-},\quad 2\le s\le 5. \tag{13}$$

The magnitude of the slopes of the class function of the p th objective is given by

$$w_{ps}^{+}=\tilde{Z}_s/\tilde{t}_{ps}^{+},\quad w_{ps}^{-}=\tilde{Z}_s/\tilde{t}_{ps}^{-},\quad 2\le s\le 5. \tag{14}$$

The convexity requirement is verified by the relationship

$$\tilde{w}_{\min}=\min_{p,s}\left\{\tilde{w}_{ps}^{+},\tilde{w}_{ps}^{-}\right\}>0, \tag{15}$$

where

$$\tilde{w}_{ps}^{+}=w_{ps}^{+}-w_{p(s-1)}^{+},\quad \tilde{w}_{ps}^{-}=w_{ps}^{-}-w_{p(s-1)}^{-},\quad w_{p1}^{+}=w_{p1}^{-}=0. \tag{16}$$

To guarantee the feasibility of the system level optimization, one way is to relax the system level consistency equality constraints using inequality constraints [16]. However, it is a delicate work to determine a rational relaxed tolerance because the feasibility and the consistency have conflicting requirements for the tolerance. That is, the more relaxed the better for the feasibility while the stricter the better for the consistency [17]. In general, the interdisciplinary consistency requirements are strengthened gradually as the bi-level optimization proceeds. This paper proposes a dynamic relaxed tolerance. When the iteration number of the bi-level optimization is $n_{iteration}$, the corresponding relaxed tolerance ε is defined as follows:

$$\varepsilon = \frac{1}{\kappa n_{iteration}} \quad (\kappa > 1) . \tag{17}$$

If the system level optimization is still infeasible, the relaxed tolerance ε is adjusted by Eq. (18).

$$\varepsilon = \varepsilon\tau \quad (\tau > 1) . \tag{18}$$

4. Subsystem optimization based on NSGA-II

The method based on NSGA-II is proposed for the subsystem optimization which has physical objectives. The subsystem objective functions include the interdisciplinary incompatibility function, physical objective functions and their variant functions. However, these objective functions have different priority levels. At the first priority level, the best individual should be in the feasible region of the subsystem. At the second priority level, the interdisciplinary incompatibility function of the best individual should be no more than the feasibility threshold. After the first and the second priority levels are achieved, physical objectives functions and their variant functions are improved.

4.1 Constraint handling

In the nondominated sorting of NSGA-II [18], an individual $\mathbf{x}_i$ is said to constrain-dominate an individual $\mathbf{x}_j$, if any of the following conditions is true:

(1) Individual $\mathbf{x}_i$ is feasible and individual $\mathbf{x}_j$ is not.
(2) Individual $\mathbf{x}_i$ and $\mathbf{x}_j$ are infeasible, but individual $\mathbf{x}_i$ has a smaller overall constraint violation.
(3) Individual $\mathbf{x}_i$ and $\mathbf{x}_j$ are feasible, and individual $\mathbf{x}_i$ dominates individual $\mathbf{x}_j$.

The overall constraint violation of an individual is given by all constraints summed together [19]. Firstly, all constraints are normalized in the following form:

$$g_i(\mathbf{x}) \le 0 . \tag{19}$$

Constraints whose values are less than zero are replaced by zero.

$$g_i(\mathbf{x_j}) = \begin{cases} 0 & g_i(\mathbf{x_j}) < 0 \\ g_i(\mathbf{x_j}) & g_i(\mathbf{x_j}) \ge 0 \end{cases} . \tag{20}$$

All constraints are assumed to be equally important. The overall constraint violation φ_{con} of individual $\mathbf{x}_j$ is expressed as the following:

$$\varphi_{con}(\mathbf{x_j}) = \sum_{i=1}^{c} g_i(\mathbf{x_j}) \tag{21}$$

where c denotes the number of subsystem constraints.

4.2 Producing an initial population at the first priority level

The individuals should be in the feasible region of the subsystem at the first priority level. Since this evolution process is irrelevant to the target values allocated by the system level, an initial population with diversity is produced in the feasible region of the subsystem. By utilizing this population, the same evolution process at the first priority level is avoided each time, which contributes to the improvement of calculation efficiency. By utilizing this population, the first priority is satisfied, which contributes to the improvement of calculation accuracy.

4.2.1 Feasibility and diversity measurements

(1) Feasibility measurement

The feasibility of an individual $\mathbf{x}_j$ can be measured by its overall constraint violation $\varphi_{con}(\mathbf{x}_j)$ given by Eq. (21). If the value of $\varphi_{con}(\mathbf{x}_j)$ is zero, then $\mathbf{x}_j$ is a feasible individual. Otherwise, $\mathbf{x}_j$ is an infeasible individual. The bigger the $\varphi_{con}(\mathbf{x}_j)$ value, the worse the feasibility of the individual $\mathbf{x}_j$. The feasibility of a population can be measured by the average constraint violation of all the individuals. The feasibility measurement φ_{fea} of a population is expressed as

$$\varphi_{fea} = \frac{\sum_{j=1}^{popsize} \varphi_{con}(\mathbf{x}_j)}{popsize} \tag{22}$$

where $popsize$ denotes the population size.

(2) Diversity measurement

If the dimension of the design vector is within three, the distribution map of the design vector can be used to represent the diversity of a population. When the dimension of the design vector is more than three, the diversity measurement φ_{div} of a population is expressed as

$$\varphi_{div} = \sum_{i=1}^{m} \frac{\Delta x_i}{x_i^u - x_i^l} \tag{23}$$

where x_i^u and x_i^l are the upper bound and lower bound of the design variable x_i respectively. Δx_i denotes the different value between maximum value and minimum value of x_i. m denotes the number of design variables.

4.2.2 Increasing the feasibility and diversity of initial population

(1) Adding random individuals

The produced population has a poor feasibility in some conditions. A certain proportion of random individuals are added in each generation except for the final generation to solve this problem. The phenomenon that all the individuals are similar is avoided during the evolution process due to the added random individuals. The final population has a good feasibility and the first priority level is satisfied.

(2) Adjusting feasibility threshold

The feasibility and diversity of the final population have conflicting requirements for the feasibility threshold of the individual, that is, the relative bigger threshold the better for the diversity, while the relative smaller threshold the better for the feasibility. The threshold is reduced gradually in the evolution process. The relative bigger threshold contributes to the diversity at the earlier stage. The relative smaller threshold contributes to the feasibility at the later stage. The threshold ϕ_{con1} is expressed as follows:

$$\phi_{con1} = T_{con}\left(\sum_{j=1}^{popsize} \varphi_{con}(\mathbf{x}_j)\right) / popsize \tag{24}$$

$$T'_{con} = 1 - \gamma \cdot gen / maxgen \quad (\gamma > 1) \tag{25}$$

$$T_{con} = \begin{cases} T'_{con} & (T'_{con} \geq 0) \\ 0 & (T'_{con} < 0) \end{cases}, \tag{26}$$

where γ is the control parameter. gen denotes the number of the generations. $maxgen$ denotes the number of maximum generations. In addition, the number of least feasible individuals n_{con} and the corresponding threshold ϕ_{con2} are set as follows:

$$n'_{con} = \lceil \alpha \cdot popsize \cdot (maxgen - gen) / maxgen \rceil \quad (\alpha < 0.1) \tag{27}$$

$$n_{con} = \begin{cases} n'_{con} & (n'_{con} \geq 0) \\ 0 & (n'_{con} < 0) \end{cases}, \tag{28}$$

$$\phi_{con2} = \begin{cases} constr(n_{con}, constrainpos) & (n_{con} > 0) \\ 0 & (n_{con} = 0) \end{cases}, \tag{29}$$

where $constr$ is the sorted population by increasing value of φ_{con}. $constrainpos$ is the nesting position of φ_{con} in the population. The bigger one between ϕ_{con1} and ϕ_{con2} is chosen as the final threshold ϕ_{con}:

$$\phi_{con} = \max\{\phi_{con1}, \phi_{con2}\} . \tag{30}$$

4.3 Setting dynamic threshold at the second priority level

The interdisciplinary incompatibility function of the best individual should be no more than the feasibility threshold at the second priority level. If the threshold is zero, only the individuals closest to the target values allocated by the system level can satisfy the second priority level. This phenomenon will hinder the physical objectives to obtain better solutions.

The proposed dynamic threshold decreases gradually as the bi-level optimization proceeds. The relative bigger threshold helps the physical objectives to obtain better solutions at the earlier stage. The threshold becomes zero and the interdisciplinary consistency requirements are ensured at the later stage. The dynamic threshold ϕ_{inc1} is defined as follows:

$$\phi_{inc1} = T_{inc}\left(\sum_{j=1}^{popsize} J_i(\mathbf{x}_j)\right) / popsize \tag{31}$$

$$T'_{inc} = 1 - SSi / SSn \tag{32}$$

$$T_{inc} = \begin{cases} T'_{inc} & (T'_{inc} \geq 0) \\ 0 & (T'_{inc} < 0) \end{cases}, \tag{33}$$

where J_i denotes the interdisciplinary incompatibility function. SSi denotes the iteration number of the bi-level optimization. SSn denotes the estimated value of the max iteration number. The number of least feasible individuals n_{inc} and the corresponding threshold ϕ_{inc2} are given as follows:

$$n'_{inc} = \lceil \beta \cdot popsize \cdot (SSn - SSi) / SSn \rceil \quad (\beta < 0.1) \tag{34}$$

$$n_{inc} = \begin{cases} n'_{inc} & (n'_{inc} \geq 0) \\ 0 & (n'_{inc} < 0) \end{cases}, \tag{35}$$

$$\phi_{inc2} = \begin{cases} incompat(n_{inc}, incompos) & (n_{inc} > 0) \\ 0 & (n_{inc} = 0) \end{cases}, \tag{36}$$

where $incompat$ is the sorted population by increasing value of J_i. $incompos$ is the nesting position of J_i in the population. The bigger one between ϕ_{inc1} and ϕ_{inc2} is chosen as the final threshold ϕ_{inc} at the second priority level:

$$\phi_{inc} = \max\{\phi_{inc1}, \phi_{inc2}\} . \tag{37}$$

4.4 Selecting a single solution from the Pareto set

Since the multiobjective subsystem has multiple solutions in the form of a Pareto set, a decision should be made to select a single solution from its Pareto set for the system level optimization. Ref. [13] gives four strategies to map a Pareto set to one system individual:

(1) A solution with the best value for any one of the objectives according to the DM's preference.
(2) A solution with the worst value for any one of the objectives according to the DM's preference.
(3) A solution chosen arbitrarily.
(4) A combination of subsystem Pareto solutions. The Pareto solutions for each subsystem are combined.

Strategy (1) is suitable for the case that one of the objectives is more important than others. Strategy (2) is suitable for the case that one objective is relatively less important. Strategy (3) is suitable for the case that all the objectives are the same important. Strategy (4) will not allow passing the value of the coupling variables from subsystems to the system level.

Since the interdisciplinary incompatibility function is more important than physical objective functions and their variant functions, strategy (1) is selected. The solution with the best value for the interdisciplinary incompatibility function is chosen.

5. Engineering example

The design of the speed reducer is a well-known problem [3, 13, 17, 19, 20]. A three-objective optimization formation is given [3, 13]. They are: minimize the total volume of the speed reducer, minimize the maximum stress in the first and second gear shaft. The formations are given in Eqs. (38)-(40), respectively.

$$\begin{aligned} min f_1(x) = {} & 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - \\ & 1.5079x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) + \\ & 0.7854(x_4x_6^2 + x_5x_7^2) \end{aligned} \quad (38)$$

$$min \;\; f_2(x) = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 1.69\times10^7}}{0.1x_6^3} \quad (39)$$

$$min \;\; f_3(x) = \frac{\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 1.575\times10^8}}{0.1x_7^3} \quad (40)$$

$$\begin{aligned} s.t. \quad & g_1 = 27/(x_1x_2^2x_3) - 1.0 \le 0 \\ & g_2 = 397.5/(x_1x_2^2x_3^2) - 1.0 \le 0 \\ & g_3 = 1.93x_4^3/(x_2x_3x_6^4) - 1.0 \le 0 \\ & g_4 = 1.93x_5^3/(x_2x_3x7^4) - 1.0 \le 0 \\ & g_5 = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 1.69\times10^7}}{180x_6^3} - 1.0 \le 0 \\ & g_6 = \frac{\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 1.575\times10^8}}{110x_7^3} - 1.0 \le 0 \\ & g_7 = x_2x_3/40 - 1.0 \le 0 \\ & g_8 = 5x_2/x_1 - 1.0 \le 0 \\ & g_9 = x_1/12x_2 - 1.0 \le 0 \\ & g_{10} = (1.5x_6 + 1.9)/x_4 - 1.0 \le 0 \\ & g_{11} = (1.1x_7 + 1.9)/x_5 - 1.0 \le 0 \\ & 2.6 \le x_1 \le 3.6 \quad 0.7 \le x_2 \le 0.8 \\ & 17 \le x_3 \le 28 \quad 7.3 \le x_4 \le 8.3 \\ & 7.3 \le x_5 \le 8.3 \quad 2.9 \le x_6 \le 3.9 \\ & 5.0 \le x_7 \le 5.5 \end{aligned}$$

where x_1 is gear face width. x_2 is teeth module. x_3 is number of teeth pinions. x_4 and x_5 are distances between bearings 1 and bearings 2, respectively. x_6 and x_7 are diameters of shaft 1 and shaft 2, respectively. g_1 is upper bound on the bending stress of the gear tooth. g_2 is upper bound on the contact stress of the gear tooth. g_3 and g_4 are upper bound on the transverse deflection of the shaft. g_5 and g_6 are upper bound on the stresses of the shaft. g_7, g_8 and g_9 are dimensional restrictions based on experience. g_{10} and g_{11} are design conditions for the shaft based on experience.

5.1 Mathematical expression

There are several reported decomposed formulations for this example [3]. We follow the decomposed formulation reported in Ref. [20]. This problem is decomposed into three subsystems. Class-1 of the LPP method is used in the system level and NSGA-II is used in the subsystem 2. The expression is shown as follows:

System level optimization:

$$\begin{aligned} & \min F(\mathbf{z}) = \sum_{p=1}^{4}\sum_{s=2}^{5} \tilde{w}_{ps}^{+} d_{ps}^{+} \quad (41) \\ & s.t. \; f_{0p}(\mathbf{z}) - d_{ps}^{+} \le t_{p(s-1)}^{+} \\ & f_{0p}(\mathbf{z}) \le t_{p5}^{+}, \\ & d_{ps}^{+} \ge 0, \\ & J_i^*(\mathbf{z}) \le \varepsilon, \\ & i = 1,2,3, \end{aligned}$$

where four objective functions $f_{01} \sim f_{04}$ are f_1, f_2, Δf_1 and Δf_2, respectively.

Subsystem 1 optimization:

$$\begin{aligned} & min J_1(\mathbf{x}_1) \quad (42) \\ & s.t. \; g_j + \Delta g_j \le 0 \quad j = 1,2,7,8,9. \end{aligned}$$

Subsystem 2 optimization:

$$\begin{aligned} & \min f_{21}(\mathbf{x}_2) = J_2(\mathbf{x}_2) \quad (43) \\ & \min f_{22}(\mathbf{x}_2) = f_3(\mathbf{x}_2) \quad (44) \\ & \min f_{23}(\mathbf{x}_2) = \Delta f_3(\mathbf{x}_2) \quad (45) \\ & s.t. \; g_j + \Delta g_j \le 0 \quad j = 1,2,4,6,7,8,9,11. \end{aligned}$$

Subsystem 3 optimization:

$$\begin{aligned} & min J_3(\mathbf{x}_3) \quad (46) \\ & s.t. \; g_j + \Delta g_j \le 0 \quad j = 1,2,3,5,7,8,9,10. \end{aligned}$$

The worst case variability in the design variable is assumed to be $\pm 1\%$. The preference of $f_{01} \sim f_{04}$ are listed in Table 1.

5.2 Producing initial population for subsystem optimization

Table 2 lists the feasibility and diversity of populations produced by three methods. (i) The strategies presented in section 4.2.2 are not used. (ii) The strategy of adding random individuals is used. (iii) The strategies of adding random individuals and adjusting feasibility threshold are used.

For the population produced by method (i), the feasibility is not satisfied and the diversity is poor. The unsatisfied feasibility means that the population is outside the feasible region. For the population produced by method (ii), the feasibility is satisfied due to the added random individuals. The diversity is increased compared with the population produced by method (i). For the population produced by method (iii), the feasibility is satisfied due to the added random individuals. The diversity is the best compared with the populations produced by method (i) and (ii) due to the adjusted feasibility threshold of the individual.

5.3 Effect analysis of initial population and dynamic threshold

The initial population produced by method (iii) in section 5.2 is used in the subsystem 2 optimization. Four cases are given to demonstrate the effectiveness of the initial population at the first priority level and the dynamic threshold at the second priority level. "Without initial population" refers to the case that the initial population is not used. "With initial population" refers to the case that the initial population is used. "Fixed threshold" refers to the case that the threshold at the second priority level is zero. "Dynamic threshold" refers to the case that the dynamic threshold at the second priority level is used. Table 3 lists the optimization results when the target value allocated by the system level is $\mathbf{z_0} = (2.0, 0.4, 9.0, 4.0, 4.5, 1.5, 3.0)$.

Referring to the results obtained from the case of "Without initial population", "Fixed threshold" and "Dynamic threshold" achieve the same optimal solutions. The overall constraint violation φ_{con} of the optimal solution is nonzero, which indicates that the first priority level is not satisfied and the second priority level is not considered.

Referring to the results obtained from the case of "With initial population", "Fixed threshold" and "Dynamic threshold" achieve distinct optimal solutions. The results obtained by "Dynamic threshold" are better with respect to f_3 and Δf_3. This is because the dynamic threshold is relatively bigger at the earlier stage of the bi-level optimization and this bigger threshold helps the physical objectives to obtain better solutions. The overall constraint violation φ_{con} of the optimal solution is zero, which indicates that the first priority level is satisfied due to the initial population. The calculation accuracy is improved compared with the results obtained from the case of "Without initial population".

5.4 Optimization results

The weighted-sum approach, preference-based approach and MOEA approach are three main methods that have been used. LPP belongs to the preference-based approach. NSGA-II belongs to the MOEA approach. The weighted-sum approach, LPP and NSGA-II are used, respectively, in the multiobjective subsystem optimization. "NSGA-II with fixed threshold" refers to the proposed method with fixed threshold at the second priority level. "NSGA-II with dynamic threshold" refers to the proposed method with dynamic threshold at the second priority level. Table 4 presents the optimization results.

Table 1. Desirable ranges of each criterion.

Objective	f_{01}	f_{02}	f_{03}	f_{04}
t_{p1}^+	3000	80	150	4
t_{p2}^+	3300	100	200	6
t_{p3}^+	3600	150	300	8
t_{p4}^+	3900	200	400	12
t_{p5}^+	4200	250	600	15

Table 2. Feasibility and diversity.

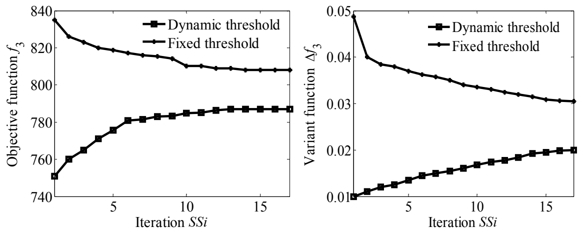
Measurement	Method (i)	Method (ii)	Method (iii)
Feasibility	0.0221	0	0
Diversity	2.6040×10^{-14}	0.4110	1.4308

Table 3. Optimization results of subsystem.

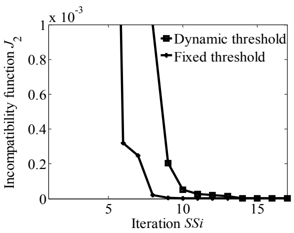
First priority level	Second priority level	J_2	f_3	Δf_3	φ_{con}
Without initial population	Fixed threshold	367.5779	806.0840	0.0277	0.0221
	Dynamic threshold	367.5779	806.0840	0.0277	0.0221
With initial population	Fixed threshold	70.7629	834.9325	0.0487	0
	Dynamic threshold	89.9516	750.7552	0.0100	0

Table 4. Robust optimization results of four different methods.

Method	f_1	Δf_1	f_2	Δf_2	f_3	Δf_3	J_2
LPP	3589.7	174.8163	76.7815	0.0021	754.7883	0.0043	0.1054
Weighted-sum	3256.9	149.7043	83.0797	0.0023	825.2615	0.0487	8.854×10^{-3}
NSGA-II with fixed threshold	3362.5	164.9392	78.0198	0.0021	808.1011	0.0305	1.494×10^{-7}
NSGA-II with dynamic threshold	3323.5	162.6952	77.8524	0.0021	787.0120	0.0200	1.474×10^{-7}



(a) Objective function f_3 (b) Variant function Δf_3



(c) Interdisciplinary incompatibility function J_2

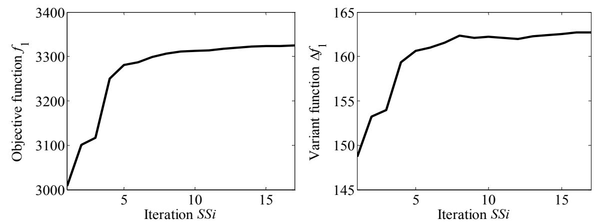
Fig. 2. Convergence plots for objective functions of subsystem level.

Referring to the results obtained from LPP, the value of the interdisciplinary incompatibility function J_2 is more than zero, which indicates that the interdisciplinary consistency is not satisfied. Referring to the results obtained from the weighted-sum approach, the value of J_2 becomes smaller than the result obtained from LPP, which results from using a bigger weight for the interdisciplinary incompatibility function. The value of J_2 obtained from both the weighted-sum and LPP approaches is not satisfied, since the interdisciplinary incompatibility function and physical objectives are on the same priority. Therefore, the weighted-sum and LPP approaches are not suitable for the multiobjective subsystem optimization.

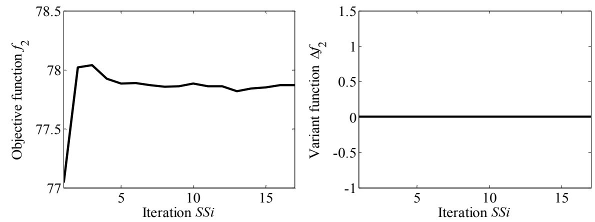
Referring to the results obtained from the proposed NSGA-II method, the value of J_2 can be seen as zero, which indicates that the interdisciplinary consistency is satisfied. The satisfied value of J_2 is obtained due to the priority setting. The objective function f_1 and its variant function Δf_1 are in the tolerable region and the desirable region respectively. The objective function f_2 and its variant function Δf_2 are both in the ideal region.

The results obtained from "NSGA-II with dynamic threshold" are better than the results obtained from "NSGA-II with fixed threshold" due to the dynamic threshold at the second priority level. At the earlier stage of the bi-level optimization, the relative bigger threshold helps to obtain better solutions with respect to f_3 and Δf_3. At the later stage, the threshold becomes zero and the interdisciplinary consistency requirements are satisfied. Fig. 2 show the convergence histories for the objective function f_3, Δf_3 and the interdisciplinary incompatibility function J_2 during the whole bi-level optimization process.

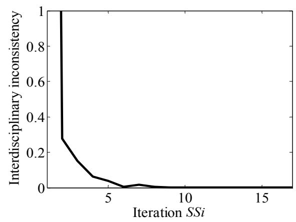
In addition, the interdisciplinary inconsistency is an important parameter during the bi-level optimization process. The



(a) Objective function f_1 (b) Variant function Δf_1



(c) Objective function f_2 (d) Variant function Δf_2



(e) Interdisciplinary inconsistency

Fig. 3. Convergence plots for objective functions of system level.

expression of the interdisciplinary inconsistency k is given based on the interdisciplinary incompatibility functions returned by all the subsystems as the following:

$$k = \sum_{i=1}^{n} J_i(\mathbf{x}_i^*, \mathbf{z}_i^*) \quad (47)$$

Fig. 3 present the convergence histories for the objective function f_1, Δf_1, f_2, Δf_2 and the interdisciplinary inconsistency k. It can be seen from Fig. 3(e) that the interdisciplinary inconsistency k decreases gradually as the bi-level optimization proceeds. Finally, the interdisciplinary inconsistency curve converges to the zero point and the interdisciplinary consistency requirements are satisfied.

6. Conclusions

A new method based on LPP and NSGA-II is proposed for the MORCO problem with multiple objectives at the system and subsystem levels. The main work focuses on the subsystem optimization with physical objectives, which has two new characteristics. On one hand, the initial population with feasibility and diversity can improve the calculation efficiency and accuracy of the subsystem optimization. On the other hand, the proposed dynamic feasibility threshold helps to obtain better solutions for the subsystem physical objectives.

An engineering example of the speed reducer is provided to illustrate the effectiveness of the proposed method. Based on the results, three conclusions can be drawn. First, the proposed method for producing an initial population is the best one compared with the other two methods, through the comparison of the produced populations on the feasibility and the diversity. Second, the calculation accuracy of the subsystem optimization is improved due to the initial population. Third, the proposed method based on LPP and NSGA-II achieves good results with respect to the subsystem physical objectives and interdisciplinary consistency compared with two other RCO methods.

Acknowledgment

This work was supported by the National High Technology Research and Development Program of China (Grant No. 2009AA04Z104).

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