

# Dissipative proportional integral observer for a class of uncertain nonlinear systems<sup>†</sup>

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# Abstract

For a class of uncertain nonlinear systems with sector-bounded nonlinearity, a proportional-integral state observer is designed in a dissipativity framework. The dissipativity ensures the bounded estimation errors against the bounded disturbance and an integral term in the observer structure can provide additional degrees of freedom in the observer design that can be used to improve the estimation performance. In this study, a dissipative condition for the proposed observer is found in terms of a linear matrix inequality (LMI). In order to specify the dissipativity, the L2 gain between the disturbance and the weighted sum of estimation errors is adopted as the supply rate in the dissipativity framework. In addition, the notion of the proposed design method is extended to the exponential dissipativity. The effectiveness of the proposed observer is demonstrated through a numerical example.

Keywords: Dissipative observer; Convex optimization; Proportional-integral observer; Robustness

# 1. Introduction

Observer problems in nonlinear systems have been studied in a variety of design approaches. For instance, several papers utilize a coordinate transformation so that the estimation error dynamics is linear in a new coordinate [1-4]. The necessary and sufficient conditions for the existence of a coordinate transformation have been established, but it is difficult to satisfy the conditions in practice. If the transformation cannot be found, sufficient conditions have been suggested depending on the Lipschitz constants or a sector-bounded constraint [5-7]. Recent research proposed observer design in the passivity framework or with the input-to-state stability property [8, 9]. All the above observers are categorized into a proportionaltype observer in the sense that they have only a proportional correction term on the output estimation error, which may lead to lack of robustness against uncertainties.

In the area of linear observers, proportional-integral-type observers have been studied. This type of observers has additional degrees of freedom in the observer design and thus can improve steady-state accuracy and estimation robustness against disturbances, unknown inputs, or modeling errors [10-12]. For example, the additional loop was used to identify unknown nonlinearities or to increase the stability margin in

the LTR design [10, 11]. Some extensive work has been done on nonlinear systems [12, 13]. However, this work is limited to a special form of nonlinearity that satisfies a triangular structure and a global Lipschitzian.

This paper proposes a proportional integral state observer in the dissipativity framework for a class of uncertain nonlinear systems with sector bounded nonlinearity. A dissipative condition for the proposed observer is found in terms of a linear matrix inequality (LMI). In order to specify the dissipativity, this paper adopts the L2 gain between the disturbance and the weighted estimation error as the supply rate in the dissipativity framework. The additional integral feedback loop is also implemented in the observer structure to improve steady-state accuracy.

# 2. Dissipative proportional integral observer

## 2.1 The scheme of proportional integral observers

Consider a class of nonlinear systems as described by

$$\dot{x} = Ax + B_u u + B_f f(C_f x) + B_w w$$

$$y = Cx + D_w w$$
(1)

where  $x \in \Re^{n \times 1}$ ,  $u \in \Re^{m \times 1}$ ,  $y \in \Re^{p \times 1}$ , and  $w \in \Re^{d \times 1}$  denote the input vector, the state vector, the output vector, and the disturbance vector, respectively. Here, the disturbance w is assumed to be bounded. The matrices  $A \in \Re^{n \times n}$ ,  $B_u \in \Re^{n \times m}$ ,  $B_f \in \Re^{n \times q}$ ,  $B_w \in \Re^{n \times (n+d)}$ ,  $C \in \Re^{m \times n}$ ,  $C_f \in \Re^{n \times n}$ , and  $D_w \in \Re^{m \times (n+d)}$  have their

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appropriate dimensions. The nonlinear term  $f(C_f x): \Re^r \to \Re^q$  represents a nonlinear drift satisfying an incremental sector bounded constraint [7, 14]. This incremental sector bounded constraint allows

$$(\varphi(\delta) + \Lambda \delta)^T (\varphi(\delta) - \Lambda \delta) \le 0 \tag{2}$$

for all  $\delta \in \Re^r$  and some symmetric positive definite matrix  $\Lambda = \Lambda^T > 0$ , where  $\phi(\delta) := f(x) - f(x - \delta)$ . This type of constraint is common in mechanical systems with actuator saturation nonlinearities [7].

A nonlinear proportional-integral observer can be designed as follows:

$$\dot{\hat{x}} = A\hat{x} + B_u u + B_f f(C_f \hat{x}) + L(y - C\hat{x}) + Kz$$
  
$$\dot{z} = y - C\hat{x}$$
(3)

where  $\hat{x} \in \Re^{n \times 1}$  and  $z \in \Re^{p \times 1}$  denote the estimated states and the integral variable. The matrices  $L \in \Re^{n \times p}$  and  $K \in \Re^{n \times p}$  represent the proportional and the integral observer gains, respectively. By subtracting Eq. (3) from Eq. (1), the error dynamics can be obtained as given by

$$\dot{e} = (A - LC)e + B_f \varphi(C_f e) - Kz + (B_w - LD_w)w$$
  
$$\dot{z} = Ce + D_w w$$
(4)

where  $e = x - \hat{x}$  represents the state estimation error, and the observer gain matrices *L* and *K* need to be selected to ensure the stability of the error dynamics. Here, substituting  $\hat{x} = x - e$  into the nonlinearity  $\varphi(C_f e)$  gives

$$\varphi(C_f e) := f(C_f x) - f(C_f \hat{x}) = f(C_f x) - f(C_f (x - e)).$$
(5)

The assumption on the incremental sector bounded constraint states that by adopting Eq. (2), the nonlinear term  $\varphi(C_f e)$  satisfies a sector constraint as follows:

$$\left(\varphi(C_f e) + \Lambda C_f e\right)^T \left(\varphi(C_f e) - \Lambda C_f e\right) \le 0 \tag{6}$$

for all  $e \in \mathfrak{R}^{n \times 1}$ . This implies that the nonlinear term  $\varphi(C_f e)$  belongs to the sector  $[-\Lambda, \Lambda]$  [7].

## 2.2 Stabilizing observer gains

Using the Lyapunov stability analysis, a sufficient condition for the stability of the PI observer is formulated into a linear matrix inequality as shown in the following theorem.

**Theorem 1**: The error dynamics equation shown in Eq. (4) with satisfying Eq. (6) is stable if there exist  $P = P^{T} > 0$ ,  $W = W^{T} > 0$ , Q, S, F, G, and  $\tau \ge 0$  subject to

$$\begin{bmatrix} A^{T}P + PA - C^{T}S - S^{T}C + \tau C_{f}^{T}\Lambda^{T}\Lambda C_{f} & PB_{f} & -Q + C^{T}W & PB_{w} - S^{T}D_{w} \\ B_{f}^{T}P & -\tau I & 0 & 0 \\ -Q^{T} + WC & 0 & 0 & WD_{w} \\ B_{w}^{T}P - D_{w}^{T}S & 0 & D_{w}^{T}W & 0 \end{bmatrix} < 0$$

$$(7)$$

and the observer gain matrices are calculated as  $L = P^{-1}S^{T}$ and  $K = P^{-1}Q$ , respectively.

**Proof**: Consider a Lyapunov function candidate for the error dynamics shown in Eq. (4):

$$V(e, z) = e^T P e + z^T W z$$
(8)

where  $P = P^{T} > 0$  and  $W = W^{T} > 0$ . The time derivative of the Lyapunov function candidate should be negative semi-definite for the stability of the error dynamics. After substituting Eq. (4) into Eq. (8), the time derivative of the function is derived as

$$\dot{V} = e^{T} [(A - LC)^{T} P + P(A - LC)]e^{-z^{T}} K^{T} P e^{-e^{T}} P K z + e^{T} C^{T} W z + z^{T} W C e^{+\varphi^{T}} B_{f}^{T} P e^{+e^{T}} P B_{f} \varphi + e^{T} P (B_{w} - LD_{w}) w + w^{T} (B_{w} - LD_{w})^{T} P e^{+z^{T}} W D_{w} w + w^{T} D_{w}^{T} W z$$

$$(9)$$

where  $\varphi := \varphi(C_f e)$ . The condition on  $\dot{V} \le 0$  and the sector constraint of Eq. (6) can be written as

$$\begin{bmatrix} e \\ \varphi \\ z \\ w \end{bmatrix}^{T} \begin{bmatrix} (A-LC)^{T}P + P(A-LC) & PB_{f} & -PK + C^{T}W & P(B_{w} - LD_{w}) \\ B_{f}^{T}P & 0 & 0 & 0 \\ -K^{T}P + WC & 0 & 0 & WD_{w} \\ (B_{w} - LD_{w})^{T}P & 0 & D_{w}^{T}W & 0 \end{bmatrix}^{T} \begin{bmatrix} e \\ \varphi \\ z \\ w \end{bmatrix} \leq 0$$
(10)

and

$$\begin{bmatrix} e \\ \varphi \end{bmatrix}^{T} \begin{bmatrix} -C_{f}^{T} \Lambda^{T} \Lambda C_{f} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} e \\ \varphi \end{bmatrix} < 0$$
(11)

By introducing the new variables,  $S = L^T P$  and Q = PK, and by using the S-procedure, Eqs. (10) and (11) hold if there exist  $\tau \ge 0$  such that

$$\begin{bmatrix} A^T P + PA - C^T S - S^T C + \tau C_f^T \Lambda^T \Lambda C_f & PB_f & -Q + C^T W & PB_w - S^T D_w \\ B_f^T P & -\tau I & 0 & 0 \\ -Q^T + WC & 0 & 0 & WD_w \\ B_w^T P - D_w^T S & 0 & D_w^T W & 0 \end{bmatrix} < 0$$

over the variables *P*, *W*, *S*,  $\tau$  and *Q* [15]. The error dynamics shown in Eq. (4) is then stable if the linear matrix inequality (LMI) of Eq. (7) is feasible. In addition, since *V* is bounded against the bounded disturbance,  $e \in L_{\infty}$ ,  $\dot{e} \in L_{\infty}$  and  $z \in L_{\infty}$ . Furthermore, since  $z = \int (y - C\hat{x}) \in L_{\infty}$ , the output estimation error goes to zero as time goes to infinity.

In addition, by post multiplying  $P^{-1}$  to both sides of the relation  $S = L^T P$  and taking the transpose, the resulting proportional observer gain L is determined as  $L = P^{-1}S^T$ . In a similar manner, from the relations of Q = PK, the other gain matrices are calculated as  $K = P^{-1}Q$ .

#### 2.3 Dissipativity in the observer structure

This section deals with the dissipativity in the PI observer

structure. Consider the following quadratic supply rate to present the dissipativity property between the disturbance and the estimation error in the input-output pair (w, (e, z)) such as

$$\varsigma(e, z, w) \coloneqq \begin{bmatrix} e \\ z \end{bmatrix}^T \Sigma \begin{bmatrix} e \\ z \end{bmatrix} + 2 \begin{bmatrix} e \\ z \end{bmatrix}^T \Pi^T w + w^T M w$$
(12)

where  $\Sigma \in \Re^{(n+p)\times(n+p)}$ ,  $\Pi \in \Re^{d\times(n+p)}$ , and  $M \in \Re^{d\times d}$  are matrices with appropriate dimensions [16]. By the definition of the dissipativity, the error dynamics shown in Eq. (4) is dissipative with the quadratic supply rate if there exists a positive definite storage function *V* such that

$$V(e,z,T) - V(e,z,0) \le \int_0^T \zeta(e,z,w) ds$$
(13)

for all  $T \ge 0$  [17]. By substituting Eq. (12) into Eq. (13) and differentiating Eq. (13), the following relation is obtained:

$$\dot{V}(e,z) \leq \begin{bmatrix} e \\ z \end{bmatrix}^T \Sigma \begin{bmatrix} e \\ z \end{bmatrix} + 2 \begin{bmatrix} e \\ z \end{bmatrix}^T \Pi^T w + w^T M w.$$
(14)

Furthermore, with the block matrices of

$$\Sigma := \begin{bmatrix} \Sigma_1 & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_2 \end{bmatrix} \text{ and } \Pi := \begin{bmatrix} \Pi_1 & \Pi_2 \end{bmatrix}$$
(15)

the following LMI condition is obtained from Eq. (14):

$$\begin{bmatrix} A^{T}P + PA - C^{T}S - S^{T}C + rC_{f}^{T}\Lambda^{T}\Lambda C_{f} - \Sigma_{1} & PB_{f} & -Q + C^{T}W - \Sigma_{12} & PB_{w} - S^{T}D_{w} - \Pi_{1}^{T} \\ B_{f}^{T}P & -rI & 0 & 0 \\ -Q^{T} + WC - \Sigma_{12}^{T} & 0 & -\Sigma_{2} & WD_{w} - \Pi_{2}^{T} \\ B_{w}^{T}P - D_{w}^{T}S - \Pi_{1} & 0 & D_{w}^{T}W - \Pi_{2} & -M \end{bmatrix} < 0$$

$$(16)$$

over the variables of *P*, *S*, *Q*, *W*, and  $\tau$ . This LMI condition ensures the dissipativity in the PI observer. That is, by solving a feasibility problem subject to the LMI condition of Eq. (16), the observer gain matrices are calculated and the observer is said to be dissipative.

As a special case, the dissipativity can be connected with a finite  $L_2$  gain. Define the  $L_2$  gain from the disturbance to the estimation error as a ratio of w and the weighted sum of e. By the definition of the *supremum* and the  $L_2$  gain, there exist a scalar  $\gamma > 0$  such that

$$\int_0^T e^T W_e^T W_e e \, dt \le \gamma^2 \int_0^T w^T w \, dt \tag{17}$$

for all  $T \ge 0$ , where the matrix  $W_e$  denotes the weighting factor [15]. Then the  $L_2$  gain is less than or equal to  $\gamma$ .

The relation between the dissipativity with the quadratic supply rate and the finite  $L_2$  gain can be obtained by replacing  $\Sigma$ ,  $\Pi$ , and M in (11) with

$$\Sigma = -\begin{bmatrix} W_e & 0\\ 0 & 0 \end{bmatrix}, \Pi = 0, M = \gamma^2$$
(18)

which means the supply rate is defined as

$$\varsigma(t) \coloneqq \gamma^2 w(t)^T w(t) - e(t)^T W_e^T W_e e(t)$$
<sup>(19)</sup>

Now, the error dynamics of Eq. (4) is said to be dissipative and finite  $L_2$  gain is stable if the following LMI condition,

$$\begin{bmatrix} A^{T}P + PA - C^{T}S - S^{T}C + \tau C_{f}^{T}A^{T}AC_{f} + W_{e}^{T}W_{e} & PB_{f} & -Q + C^{T}W & PB_{w} - S^{T}D_{w} \\ B_{f}^{T}P & -\tau I & 0 & 0 \\ -Q^{T} + WC & 0 & 0 & WD_{w} \\ B_{w}^{T}P - D_{w}^{T}S & 0 & D_{w}^{T}W & -\rho I \end{bmatrix}^{2},$$
(20)

is satisfied over the variables *P*, *S*, *Q*, *W*,  $\tau$  and  $\rho$  where  $\rho := \gamma^2$ .

### 3. Exponentially dissipative PI observer design

This section extends the notion of the dissipativity to the exponential dissipativity [17]. The following definition addresses the concept of the exponential dissipativity.

**Definition 1**: The error dynamics of Eq. (4) is said to be *exponentially dissipative* with the supply rate expressed in Eq. (12) if there exists a positive definite *V* such that

$$V(e,z,T) - V(e,z,0) \le \int_0^T e^{\varepsilon s} \zeta(s) \, ds \tag{21}$$

for all  $T \ge 0$ , where the scalar,  $\varepsilon$ , denotes the rate of dissipativity.

This definition implies that the exponential dissipativity with the supply rate expressed in Eq. (19) can be expressed as

$$\frac{d}{dt}V + \varepsilon V + e^T W_e^T W_e e - \gamma^2 w^T w \le 0$$
(22)

for all *t*, *e*, *z* and *w* with V(0) = 0. By solving Eq. (22) with respect to the Lyapunov function candidate of Eq. (8), an LMI condition that addresses the exponential dissipitivity can be obtained as the following corollary.

**Corollary 1**: For a given positive scalar  $\varepsilon$  and the weighting matrix  $W_e$ , the error dynamics shown in Eq. (4) is exponentially dissipative and finite  $L_2$  gain stable against the disturbances if the following LMI,

$$\begin{bmatrix} A^{T}P + PA - C^{T}S - S^{T}C + \varepsilon P + \tau C_{f}^{T}\Lambda^{T}\Lambda C_{f} + W_{e}^{T}W & PB_{f} - Q + C^{T}W & PB_{w} - S^{T}D_{w} \\ B_{f}^{T}P & -\tau I & 0 & 0 \\ -Q^{T} + WC & 0 & \varepsilon W & WD_{w} \\ B_{w}^{T}P - D_{w}^{T}S & 0 & D_{w}^{T}W & -\rho I \end{bmatrix} < 0$$

$$(23)$$

is feasible over the variables  $P, S, Q, W, \tau$  and  $\rho$ .

Then the proposed PI observer with the exponential dissipitivity and the finite  $L_2$  gain against the disturbance is designed as follows:

$$\dot{\hat{x}} = A\hat{x} + B_{u}u + B_{f}f(C_{f}\hat{x}) + P^{-1}S^{T}(y - C\hat{x}) + P^{-1}Q \int (y - C\hat{x})$$
(24)

where P, S, and Q are the feasible solutions of Eq. (23). In addition, from Eq. (22), it is rewritten as

$$\dot{V} \leq -(\varepsilon \lambda_{\min}(P) + \lambda_{\min}(W_e)^2) |e|^2 - \varepsilon \lambda_{\min}(W) |z|^2 + \gamma^2 |w|^2 \quad (25)$$

where  $\lambda_{\min}(.)$  is the minimum eigenvalue. Furthermore, for  $|w| \leq \varepsilon \lambda_{\min}(W) / \gamma^2 \cdot |z|$ ,

$$\dot{V} \le -(\varepsilon \lambda_{\min}(P) + \lambda_{\min}(W_e)^2) |e|^2$$
(26)

Since  $e \in L_{\infty}$  and  $\dot{e} \in L_{\infty}$ ,  $\ddot{V}$  is finite. Then Barbalat's lemma states that  $\dot{V}$  goes to zero and thus the state estimation error e goes to zero as time goes to infinity.

## 4. Example

In this section, a numerical example is introduced to demonstrate the effectiveness of the proposed PI observer. Consider a flexible joint robot arm presented in Ref. [7]. The statespace model in Eq. (1) can be expressed with the system matrices of

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, B_u = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, B_f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.33 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, C_f = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix},$$

and the nonlinearity  $f(x) = \sin(x)$  satisfies the incremental sector bound condition shown in Eq. (6) with  $\Lambda = 1$ . The disturbance vector is a biased periodical disturbance,  $w = 0.5\sin(10t)+0.5$ , which can make the estimation error biased. The corresponding distribution matrices are set as  $B_w = \begin{bmatrix} I_{4\times4} & 0_{4\times2} \end{bmatrix} B_w = \begin{bmatrix} I_{4\times4} & 0_{4\times2} \end{bmatrix}$  and  $D_w = \begin{bmatrix} 0_{2\times6} \end{bmatrix}$  in this system.

In order to demonstrate the estimation performance, the proposed PI observer is designed as described in Section III. The design parameters are selected as  $W_e = \text{diag}([0.1 \ 0.1 \ 1 \ 1])$  and  $\varepsilon = 0$ , respectively. By solving the LMI shown in Corollary 1, the observer gain matrices are calculated as follows:

L =	1.4299	-9.7013 5.3721	1	2.0329	0.0430
	1.8189	22.3045	and $K =$	0.0178	0.4259
	13.8650	27.9781			-0.9269

The estimation performance of the proposed PI observer is illustrated in Fig. 1(a) and is compared with a proportionaltype observer in Fig. 1(b), where the P observer is designed by



(a) Estimated state variables



Fig. 1. The estimation results.

setting K = 0 in the proposed PI observer structure in a way that it has only a proportional correction term. The simulation shows that the estimated result by the proposed observer follows the true values very well, while the P observer makes an offset in steady-state estimation error. The results demonstrate that the proposed dissipative PI observer is very effective in improving steady-state accuracy and in eliminating biased disturbances due to the dissipative property and the integral feedback loop.

#### 5. Conclusions

In this study, for a class of nonlinear systems with uncertainties, a new formulation of nonlinear observers is proposed with including the integral feedback loop of the observation error. The concepts of the dissipativity and the exponential dissipativity are introduced to establish the boundedness of the estimation error. The quadratic supply rate in the dissipativity framework is defined as the finite L2 gain between disturbances and the weighted sum of estimation errors. A convex optimization problem subject to a linear matrix inequality is constructed for the dissipativity property and the observer gain matrices are calculated from the feasible solutions. The simulation results show that the proposed observer is effective in improving steady-state accuracy due to the dissipativity property and the integral feedback correction term.

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