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A dead reckoning localization system for mobile robots using inertial sensors and wheel revolution encoding[†]

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Abstract

Inertial navigation systems (INS) are composed of inertial sensors, such as accelerometers and gyroscopes. An INS updates its orientation and position automatically; it has an acceptable stability over the short term, however this stability deteriorates over time. Odometry, used to estimate the position of a mobile robot, employs encoders attached to the robot's wheels. However, errors occur caused by the integrative nature of the rotating speed and the slippage between the wheel and the ground. In this paper, we discuss mobile robot position estimation without using external signals in indoor environments. In order to achieve optimal solutions, a Kalman filter that estimates the orientation and velocity of mobile robots has been designed. The proposed system combines INS and odometry and delivers more accurate position information than standalone odometry.

Keywords: Positioning; Localization; Dead reckoning; INS; Inertial sensor; Compass sensor; Accelerometer; Gyroscope; Odometry; Encoder; Mobile robot; Kalman filter; Orientation compensation

1. Introduction

A mobile robot is an automatic machine that navigates in a given environment and recognizes its surroundings using many sensors. The study of mobile robots has developed rapidly in various fields, such as housework support, elder assistance, education, medical care, national defense, etc. The localization of a mobile robot in order to achieve autonomous movement is an important technique and is currently an important research field. There are two general methods used for localization: relative positioning and absolute positioning [1]. Relative positioning, also known as dead reckoning, evaluates the position of the mobile robot by using its velocity and yaw angles measured by encoders attached to the robot's wheels or by inertial sensors [1-8]. Absolute positioning evaluates the position of the mobile robot by using external distance measuring systems.

Dead reckoning estimates a relative position from the initial starting point information. Generally, it uses an inertial measurement unit (IMU) or a control variable, such as an encoder, and so does not depend on external signals [1-12]. Therefore dead reckoning has advantages in being simple, low cost, and has an easier time in estimating the position in real time compared to absolute positioning. In order to measure the position, the absolute positioning method uses the global positioning system (GPS), ultrasonic local positioning systems, infrared network systems, radio frequency identification (RFID) systems, etc. [1, 4]. GPS cannot be used indoors and has a slow update rate [1, 13]. The ultrasonic local positioning systems and the infrared network systems are low cost, small, and are easy to interface [1, 12-17]. However, these methods cannot measure far distances, require additional installation and have a difficulty in accuracy due to signal interference. RFID requires additional equipment and has a high cost. These absolute positioning methods have an advantage in that they do not accumulate positioning errors, but the overall positioning errors are relatively big.

In this paper, we discuss the position estimation of mobile robots in an indoor environment, such as a building or a factory. We assume that the mobile robot travels on a flat, mostly level surface. When the mobile robot travels, the position and the yaw angle of the mobile robot can be estimated by encoders attached to the robot's wheels. This method is known as odometry [1-8]. The encoders measure the wheel's angular rate of change. The position and yaw angle of the mobile robot are calculated by the rotary angle, and the diameter of the wheel, and the body width of the mobile robot. When using odometry, unbounded errors occur due to slippage, mismatches in the system parameters, measurement inaccuracies, and noise from the encoder signals [1-8]. Although these er-

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rors are relatively small, they do accumulate, caused by position and yaw angle errors, over the long run. The inclination of the ground is also a source for errors. These errors can be reduced by using attitude compensation. An INS updates the orientation and position information automatically without external signals. The INS, which consists of an IMU and a navigation computer, provides the position and orientation of the mobile robot at a high rate, typically 100 times per second [9-11]. If the mobile robot's initial information is known, the position and orientation are determined by the integration of the accelerations and the angular rates [9-11]. However, low frequency noise and sensor biases are amplified due to the integrative nature of the system [9, 18-20]. That is, INS offers a good short term stability but has a poor long term stability. Therefore, an INS without additional external signals has unbounded position and orientation errors.

By themselves, standalone odometry and INS are not suitable for dead reckoning for over long periods of time due to the accumulation errors. In this paper, we combine the odometry and the INS in order to reduce the accumulation errors found in the dead reckoning. Although both INS and odometry have accumulation errors, the integration of these two systems will reduce these errors to an acceptable level. In order to achieve an optimal integrated system, a Kalman filter will be designed and used [21]. The main idea is same as followings: The position of the mobile robot is estimated by the velocity and orientation that are calculated by tri-axial accelerometers and tri-axial gyroscopes for every 20 Hz period (50 ms). In order to estimate the orientation, for every 4 Hz period (250 ms), the roll and the pitch are updated using accelerometer, the yaw angle rate is updated using odometer, and the yaw angle is updated using the magnetic sensor. In order to estimate the position, the odometer refines the velocity measurement. The aim of this paper is to extend the period of travelling without needing any external position data from an absolute positioning system.

2. The navigation system

The purpose of the proposed system that combines the INS and odometry is to estimate the orientation and position of the mobile robot with only a small amount of errors, using the information from two systems that have different features. This section presents the fundamental equations of the navigation system and describes the proposed positioning algorithm.

2.1 The inertial navigation system

The INS consists of an IMU and a navigation computer. The IMU is an assembly of the inertial sensors, which include tri-axial accelerometers, tri-axial gyroscopes, and at least dual axial magnetic sensors [9-11]. The navigation computer operates using an algorithm that estimates the orientation, velocity and position. The tri-axial accelerometers measure the absolute three dimension acceleration with respect to the body frame. The inclination of the mobile robot is evaluated with three orthogonal accelerometers because the tri-axial accelerometers provide very accurate information when the mobile robot is stationary. In addition, single and double integration of the accelerations provide the velocity and position, respectively. The three orthogonal gyroscopes provide the angular rates regarding the three axes. The integration of the angular rates provides the orientation of the mobile robot. In addition the integration of the tri-axial gyroscopes provides the orientation not only when the mobile robot is stationary, but also when it is moving.

2.1.1 The initial alignment algorithm

Dead reckoning calculates a relative position from the initial position information and the initial orientation information of the mobile robot. The initial alignment process is used to calculate the initial orientation from the outputs of the inertial sensors when the mobile robot is stationary. We can assume that the starting point is the origin. The initial orientation can be calculated with three orthogonal accelerometers and dual magnetic sensors.

The acceleration ($\mathbf{f}^b$) on the body frame is measured at a complete standstill in the following manner:

$$\mathbf{f}^b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \mathbf{C}_n^b \mathbf{f}^n = \mathbf{C}_n^b \begin{bmatrix} 0 \\ 0 \\ -g^n \end{bmatrix} = \begin{bmatrix} g^n \sin\theta \\ -g^n \cos\theta \sin\phi \\ -g^n \cos\theta \cos\phi \end{bmatrix} \tag{1}$$

where $\mathbf{C}_n^b$ is the transformation matrix from the navigation frame to the body frame. $\mathbf{f}^n$ is the acceleration on the navigation frame and g^n is the gravity. ϕ and θ denote the roll angle and the pitch angle, respectively. From Eq. (1), the roll angle and pitch angle become:

$$\phi = \tan^{-1}\left(\frac{-g^n \cos\theta \sin\phi}{-g^n \cos\theta \cos\phi}\right) = \tan^{-1}\left(\frac{f_y}{f_z}\right), \tag{2}$$

$$\theta = \tan^{-1}\left(\frac{g^n \sin\theta}{g^n \cos\theta}\right) = \tan^{-1}\left(\frac{f_x}{\sqrt{f_y^{\,2} + f_z^{\,2}}}\right). \tag{3}$$

The yaw angle (φ) may be obtained from the earth rotation angular velocity when stationary. However, it is quite difficult to measure the earth's rotation angular velocity because it demands gyroscopes with very high resolutions in order to measure this very small angular acceleration value.

The magnetic sensor measures the magnitude of the earth's magnetic field using a magneto-resistive element. The compass sensor is composed of two magnetic sensors with an orthogonal orientation that sense the horizontal components of the earth's magnetic field. The compass sensor can therefore measure the yaw angle. The dual axial magnetic fields (H_{ex}, H_{ey}) that are measured by the dual magnetic sensors provide the magnetic north angle (α) in the following manner:

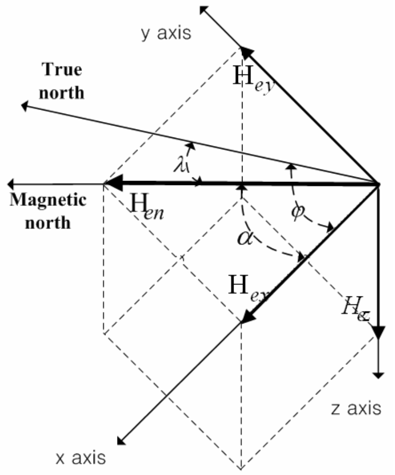


Fig. 1. The earth magnetic field vector.

$$\alpha = \tan^{-1}\left(\frac{H_{ey}}{H_{ex}}\right). \tag{4}$$

There is a difference between true north and magnetic north. This angular difference is known as the declination angle (λ). The declination varies from 0° to 30° in most populated regions of the world. These declination values change slightly over time as the earth's tectonic plate shift. Therefore, the actual declination value is different in each location. The yaw angle is a right-handed rotation about true north. Therefore the yaw angle is determined by:

$$\lambda = \varphi - \alpha, \ \varphi = \lambda + \alpha \ . \tag{5}$$

Fig. 1 depicts the earth magnetic field vector and the relationship between magnetic north and true north.

2.1.2 The bias calibration algorithm

The initial alignment process determines the initial orientation of the mobile robot. The initial alignment is very important because the dead reckoning algorithm uses this initial orientation in order to update its attitude and position [1, 9-11]. The accelerometers and gyroscopes have bias errors because of sensor misalignment, sensitivity and offset. In initial alignment, the orientation is calculated by the tri-accelerometers. When the mobile robot is moving, the orientation is calculated by the integrations of the angular rates that are measured by the tri-gyroscopes. Also the velocity and position are calculated by single and double integrations of the accelerations, respectively. Therefore, the biases of the accelerometer and gyroscope will increase the errors of the orientation and the position. Hence, the outputs of accelerometer and gyroscope must be calibrated in applications that require high accuracy, such as a mobile robot.

In principle, the inclination of the mobile robot platform is determined from the tri-accelerometers, however the unknown biases of the tri-accelerometers affect the accurate measurement of the inclination angle. In order to reduce the unknown biases of the tri-accelerometers, we will use dual-axial inclinometers. Two orthogonally mounted inclinometers measure the small deviations of the mobile robot platform at up to $\pm 30°$ from the horizontal plane (x-y plane). The inclination information that is provided by the dual-axial inclinometers is used to cancel the biases on each axis of the accelerometer. Unfortunately, this inclination information is only useful when the mobile robot is stationary, since inclinometers are inherently sensitive to acceleration. Therefore, the accelerometer biases must be determined before attempting to estimate the orientation, velocity and position while the mobile robot is moving.

When the mobile robot is stationary, an algorithm that compensates the biases of the tri-accelerometers is the following. The compass sensor measures the yaw angle and the dual-axial inclinometers measure the roll and pitch angle. We will assume that these angles (yaw, roll and pitch) are the real orientation of the mobile robot. A transformation matrix ($\mathbf{C}_b^n$) that shows the relationship between the body frame and the navigation frame becomes Eq. (6) by using these angles.

$$\mathbf{C}_b^n = \begin{bmatrix} c\varphi c\theta & c\varphi s\theta s\phi - s\varphi c\phi & c\varphi s\theta c\phi + s\varphi s\phi \\ s\varphi c\theta & s\varphi s\theta s\phi + c\varphi c\phi & s\varphi s\theta c\phi - c\varphi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{6}$$

where $c = \cos$ and $s = \sin$. The relationship between the acceleration in the body frame and the acceleration in the navigation frame is:

$$\mathbf{f}^n = \mathbf{C}_b^n \mathbf{f}^b \ . \tag{7}$$

Since the mobile robot is stationary, the accelerations found in the navigation frame are only from the gravity vector. That is, the accelerations in the north and east direction are zero. The accelerations that are transformed with the navigation frame become the accelerometer biases excluding the gravity vector. In order to compensate for the biases, the outputs of the compass sensor, dual-axial inclinometers and the tri-axial accelerometers are sampled for 10 seconds; the sampled data are then averaged and are applied in the following algorithm.

$$\mathbf{f}^n = \begin{bmatrix} f_{N_{measure}} \\ f_{E_{measure}} \\ f_{D_{measure}} \end{bmatrix} = \begin{bmatrix} f_{N_{bias}} \\ f_{E_{bias}} \\ -g^n + f_{D_{bias}} \end{bmatrix}, \tag{8}$$

$$\mathbf{f}^n{}_{bias} = \begin{bmatrix} f_{N_{bias}} \\ f_{E_{bias}} \\ f_{D_{bias}} \end{bmatrix} = \begin{bmatrix} f_{N_{measure}} \\ f_{E_{measure}} \\ f_{D_{measure}} + g^n \end{bmatrix}, \tag{9}$$

$$\hat{\mathbf{f}}^n = \mathbf{f}^n - \mathbf{f}^n{}_{bias} \tag{10}$$

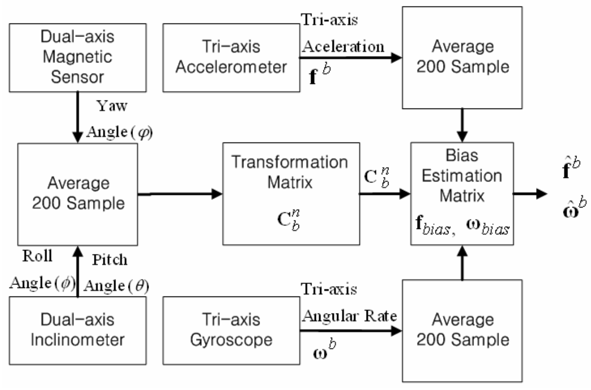


Fig. 2. The block diagram of the bias estimations.

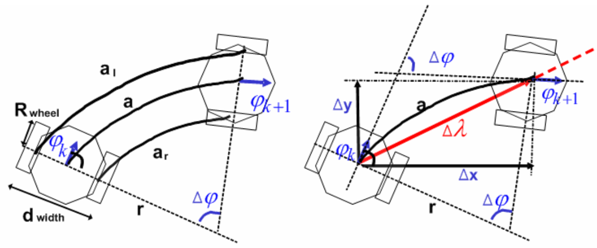


Fig. 3. The movement prediction using rotary angles.

where $\hat{\mathbf{f}}^n$ is the acceleration that removes the biases from the navigation frame. After estimating the biases in the navigation frame, the measured accelerations in the body frame are evaluated using

$$\mathbf{f}^b{}_{bias} = \mathbf{C}_n^b \mathbf{f}^n{}_{bias} , \tag{11}$$

$$\hat{\mathbf{f}}^b = \mathbf{f}^b - \mathbf{f}^b{}_{bias} . \tag{12}$$

In Eq. (11), $\mathbf{C}_n^b = \left[\mathbf{C}_b^n\right]^{-1} = \left[\mathbf{C}_b^n\right]^T$. The '-1' superscript indicates an inverse matrix and the 'T' superscript indicates a transpose matrix. $\hat{\mathbf{f}}^b$ is the acceleration that removes the biases from the body frame. The inverse matrix of the transformation matrix is equal to the transpose matrix, since the transformation matrix is orthogonal.

The tri-gyroscopes provide the three dimension angular rate with respect to the body frame. Although the angular rate is reliable over long periods of time, it must be integrated in order to provide an absolute orientation measurement. Therefore, even small errors in the angular rate generate unbounded errors. When the mobile robot is stationary, the angular rates in the body frame must be zero. Therefore, the measured angular rates are bias errors. In order to compensate for the biases, the angular rates are sampled for 10 seconds; the sampled data are then averaged. Fig. 2 shows the block diagram of the proposed bias estimation algorithm.

$$\boldsymbol{\omega}^b = \boldsymbol{\omega}^b{}_{bias} = \left[\omega_{x_{bias}} \quad \omega_{y_{bias}} \quad \omega_{z_{bias}}\right]^T , \tag{13}$$

$$\hat{\boldsymbol{\omega}}^b = \boldsymbol{\omega}^b - \boldsymbol{\omega}^b{}_{bias} \tag{14}$$

where $\hat{\boldsymbol{\omega}}^b$ is the angular rate vector that removes the biases in the body frame.

2.1.3 The position algorithm

Through Newton's law of motion, the velocity of the mobile robot is calculated by using a single integration of the acceleration. The velocity at one time-step from the present will be equal to the present velocity plus the commanded acceleration multiplied by the measurement period (Δt). The position is calculated by using a single integration of the velocity.

$$\mathbf{v}_{k+1} = \mathbf{v}_k + \mathbf{f}_k \Delta t , \tag{15}$$

$$\mathbf{P}_{k+1} = \mathbf{P}_k + \mathbf{v}_k \Delta t . \tag{16}$$

After the measured acceleration transforms the body frame into the navigation frame, by applying the acceleration of the gravity, the equation of velocity becomes:

$$\mathbf{v}_{k+1}^n = \mathbf{v}_k^n + \left(\mathbf{C}_b^n \mathbf{f}_k^b + \mathbf{g}^n\right)\Delta t \tag{17}$$

where $\mathbf{v}_k^n = \left[v_{N_k} \quad v_{E_k} \quad v_{D_k}\right]^T$ denotes the velocity vector on the navigation frame at time k. The position of the mobile robot becomes:

$$\mathbf{P}_{k+1}^n = \mathbf{P}_k^n + \mathbf{v}_k^n \Delta t + \frac{1}{2}\left(\mathbf{C}_b^n \mathbf{f}_k^b + \mathbf{g}^n\right)\Delta t^2 \tag{18}$$

where $\mathbf{P}_k^n = \left[P_{N_k} \quad P_{E_k} \quad P_{D_k}\right]^T$ denotes the position vector on the navigation frame at time k.

2.2 The odometry system

The rotor encoder is the system that converts the angular rate of the rotor into a digital signal. Generally, the rotor encoders are attached onto the wheels of the mobile robot; the wheel's rotary angle is measured by the encoder. The rotor encoder generates N pulses while the wheel rotates 360 degrees. If the measured pulses are M counts, each wheel's rotary angle becomes:

$$\eta_{k,l} = \frac{M_l}{N} \times 2\pi, \quad \eta_{k,r} = \frac{M_r}{N} \times 2\pi \tag{19}$$

where $\eta_{k,l}$ and $\eta_{k,r}$ are the left and right wheel's rotary angles in radians, respectively. M_l and M_r are the measured pulses on left and right encoders.

The velocity, position and yaw angle of the mobile robot are estimated by using each wheel's rotary angle. Fig. 3 shows the movement prediction of the mobile robot from the wheels' rotary angles. The mobile robot's travel distance a_k can be expressed in terms of its wheel's radius (R_{wheel}), and each wheel's rotary angle.

$$a_{k,l} = R_{wheel}\eta_{k,l}, \quad a_{k,r} = R_{wheel}\eta_{k,r}, \tag{20}$$

$$a_k = \frac{a_{k,l} + a_{k,r}}{2}. \tag{21}$$

The mobile robot's yaw angle rate ($\Delta\varphi_k$) is calculated by the width of the robot (d_{width}) and the distance travelled by each wheel.

$$\Delta\varphi_k = \frac{a_{k,l} - a_{k,r}}{d_{width}} \tag{22}$$

In this case, the robot's rotation radius (r_k) is:

$$r_k = \frac{a_k}{\Delta\varphi_k}. \tag{23}$$

According to the cosine law, the mobile robot's position rate ($\Delta\lambda$) can be expressed as:

$$\Delta\lambda_k^{\,2} = r_k^{\,2} + r_k^{\,2} - 2r_k^{\,2}\cos\Delta\varphi_k = 2(1-\cos\Delta\varphi_k)r_k^{\,2}. \tag{24}$$

If the robot moves in a straight line, the yaw angle rate will be zero. In this case, the rotation radius will become arbitrarily large, and the term in the parenthesis Eq. (24) will become zero. Therefore, Eq. (24) must be expanded by using a Taylor series as:

$$\begin{aligned}\Delta\lambda_k^{\,2} &= 2\left[1-\left(1-\frac{\Delta\varphi_k^{\,2}}{2!}+\frac{\Delta\varphi_k^{\,4}}{4!}-\frac{\Delta\varphi_k^{\,6}}{6!}+\cdots\right)\right]\frac{a_k^{\,2}}{\Delta\varphi_k^{\,2}} \\ &= 2a_k^{\,2}\left(\frac{1}{2!}-\frac{\Delta\varphi_k^{\,2}}{4!}+\frac{\Delta\varphi_k^{\,4}}{6!}-\frac{\Delta\varphi_k^{\,6}}{8!}+\cdots\right).\end{aligned} \tag{25}$$

The following is the mobile robot's position rate transformed into the navigation frame.

$$\begin{aligned}\Delta P_{N_k} &= \Delta\lambda_k \cos\left(\varphi_{k-1}+\frac{\Delta\varphi_k}{2}\right), \\ \Delta P_{E_k} &= \Delta\lambda_k \sin\left(\varphi_{k-1}+\frac{\Delta\varphi_k}{2}\right)\end{aligned} \tag{26}$$

If roll and pitch are experienced by the mobile robot, the position rate that compensates the attitude is:

$$\begin{aligned}\Delta\hat{P}_{N_k} &= \Delta P_{N_k}\cos\theta + \Delta P_{E_k}\sin\phi\cos\theta, \\ \Delta\hat{P}_{E_k} &= \Delta P_{E_k}\cos\phi.\end{aligned} \tag{27}$$

The mobile robot's position and yaw direction is defined as:

$$P_{N_{k+1}} = P_{N_k} + \Delta\hat{P}_{N_k}, \quad P_{E_{k+1}} = P_{E_k} + \Delta\hat{P}_{E_k}, \tag{28}$$

$$\varphi_{k+1} = \varphi_k + \Delta\varphi_k. \tag{29}$$

In this case, the velocity is a differential of the mobile robot's quantity of change in the position.

$$v_{N_k} = \frac{\Delta\hat{P}_{N_k}}{\Delta t}, \quad v_{E_k} = \frac{\Delta\hat{P}_{E_k}}{\Delta t} \tag{30}$$

3. Error compensation using the Kalman filter

Both positioning systems using encoders and that using inertial sensors have accumulative errors, but caused by different reasons. In the method using inertial sensors, integration of the outputs is the cause of the accumulated errors. In the method using encoders, systematic and nonsystematic errors are accumulated through calculation. We design a linear Kalman filter to combine these features. The Kalman filter is known to be the most ideal filter to estimate state variables in a dynamic system [19, 21].

3.1 The orientation compensation

In the initial alignment algorithm, the orientation of the mobile robot is calculated by the tri-accelerometers and the compass sensor. If the mobile robot is moving, it will have accelerated motion, that is, the tri-accelerometer will measure not only the acceleration of the gravity but also the acceleration motion of the mobile robot. In this case, the attitude has a low accuracy because the roll and pitch angles are measured by the tri-accelerometers [10, 11]. Similarly, the compass sensor, which measures the yaw angle, has errors, since small magnetic field changes can be caused by metals found in the external environment [1]. Therefore, when the mobile robot is moving, it is difficult to measure the orientation of the mobile robot using the initial alignment algorithm.

The tri-gyroscopes provide the angular rate in the body frame. The tri-gyroscopes are useful for measuring the orientation of the mobile robot while the mobile robot is moving. However the orientation must be integrated using the angular rates; the gyroscope has drift and random walk noise which can effect these integration. So the orientation estimation using only gyroscopes generate accumulated orientation errors.

In odometry, the yaw angle rate is calculated by Eq. (22). However, the yaw angle rate is inaccurate with an unbounded accumulation of systematic and nonsystematic errors [1-8].

We design the linear Kalman filter to estimate the orientation. The tri-gyroscopes are used as a system input variables because the tri-gyroscopes have fast response and less error covariance, but the error is accumulative. The tri-accelerometers, the compass sensor and yaw angle rate from odometry are used as a measurement variable because these outputs have not accumulated errors, but have big error covariance.

3.2 The position compensation

Dead reckoning is a common method used to predict the position of a mobile robot by internal sensors, generally iner-

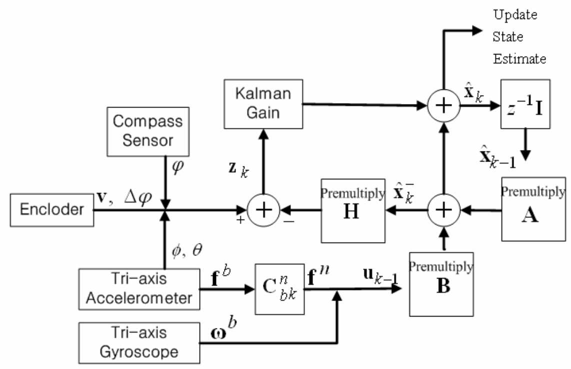


Fig. 4. The block diagram of the Kalman filter.

tial sensors, and control variables, such as the encoder. The position estimation obtained by dead reckoning has an acceptable accuracy over the short term, however it has unbounded errors over the long term. In this study, we use inertial sensors and encoders attached to the robot's wheels for positioning. The positioning method using inertial sensors must integrate the measured tri-accelerations. This integration increases the unbounded errors in the velocity and position of the mobile robot [9, 11]. In the positioning method using the encoders, the velocity and position of the mobile robot is calculated by the angular rate of rotor and the diameter of the wheel. However, unbounded errors occur due to the measuring errors found in the encoders, i.e. the mechanical design defects of the mobile robots, the slippage between the wheels and ground, etc. [1-8].

The accurate positioning of a mobile robot using either the inertial sensors or the encoders is difficult to achieve due to the unbounded errors found in each system. Although they both have unbounded velocity and position errors, a coupled system will reduce the error to an acceptable level. The tri-accelerometers are suitable for a system input variables because the tri-accelerations offer fast response and good short term stability, but have big accumulated errors due to a double integration. The outputs of orientation compensated odometry have less error covariance but have slow response (about five times per second). Therefore the outputs of orientation compensated odometry are suitable for a measurement variable in the linear Kalman filter.

3.3 The Kalman filter

Fig. 4 shows the block diagram of the Kalman filter used for dead reckoning. The linear stochastic difference equations for the Kalman filter in this case are:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_{k-1}, \tag{31}$$

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \tag{32}$$

where $\mathbf{x}_k = \begin{bmatrix} v^n_{N_k} & v^n_{E_k} & \phi_k & \theta_k & \varphi_k & \Delta\varphi_k \end{bmatrix}^T$ is the state variable at time k and are divided into the velocity, orientation and yaw angle rates of the mobile robot. The random variables $\mathbf{w}_k$ and $\mathbf{v}_k$ represent the process and measurement noise, respectively. The specific equations for the time update (predict) and the measurement update (estimate) are [21]:

$$\hat{\mathbf{x}}^-_k = \mathbf{A}\hat{\mathbf{x}}_{k-1} + \mathbf{B}\mathbf{u}_{k-1}, \tag{33}$$

$$\mathbf{K}^-_k = \mathbf{A}\mathbf{K}_{k-1}\mathbf{A}^T + \mathbf{Q}, \tag{34}$$

$$\mathbf{G}_k = \mathbf{K}^-_k\mathbf{H}^T\left(\mathbf{H}\mathbf{K}^-_k\mathbf{H}^T + \mathbf{R}\right)^{-1}, \tag{35}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}^-_k + \mathbf{G}_k\left(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}^-_k\right), \tag{36}$$

$$\mathbf{K}_k = \left(\mathbf{I} - \mathbf{G}_k\mathbf{H}\right)\mathbf{K}^-_k \tag{37}$$

where $\hat{\mathbf{x}}_k$ denotes the estimated state variable and $\hat{\mathbf{x}}^-_k$ denotes the predicted state variable. $\mathbf{K}_k$ denotes the estimation error covariance and $\mathbf{K}^-_k$ denotes the prediction error covariance. $\mathbf{G}_k$ denotes the Kalman gain.

The estimated and predicted state variables are a linear combination. We assume that the correlations between the velocity and the orientation are zero and that the system matrix is a unit matrix.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{38}$$

The system input variables are the outputs of the accelerometer and the outputs of gyroscope. The correlations of each input are zero. The input variable and state variable are only related to the measurement period and the transformation matrix.

$$\mathbf{u} = \begin{bmatrix} f^n_{N_k} & f^n_{E_k} & \omega^b_{x_k} & \omega^b_{y_k} & \omega^b_{z_k} & \omega^b_{z_k} \end{bmatrix}^T, \tag{39}$$

$$\mathbf{B} = \Delta t \times \mathbf{I}_{6\mathrm{x}6} \tag{40}$$

where $\begin{bmatrix} f^n_{N_k} & f^n_{E_k} & f^n_{D_k} \end{bmatrix}^T = \mathbf{C}^n_b\begin{bmatrix} f^b_{x_k} & f^b_{y_k} & f^b_{z_k} \end{bmatrix}^T$.

The measurement variable and the state variable are a linear combination. We assume that the correlations of the measurement variable are zero and that the measurement matrix is a unit matrix.

$$\mathbf{H} = \mathbf{I}_{6\mathrm{x}6}, \tag{41}$$

$$\mathbf{z}_k = \begin{bmatrix} v_{N_k}\big|_o & v_{E_k}\big|_o & \phi_k\big|_a & \theta_k\big|_a & \varphi_k\big|_c & \Delta\varphi_k\big|_o \end{bmatrix}^T = \mathbf{H}\mathbf{x}_k \tag{42}$$

where the 'o' subscript indicates the calculated values from the odometry and the 'a' subscript indicates the calculated values from Eqs. (2) and (3). The 'c' subscript is the output from the compass sensor.

The process noise covariance is the error covariance that is measured from the inertial sensors. If we assume that the correlations of inertial sensors are zero, the process noise covariance is the following. The white noise of velocity is related to white noise of acceleration and the measurement period. If the white noise of acceleration in the navigation frame is N_f, then the noise covariance of velocity in the navigation frame is:

$$Q_v = \mathbf{E}\left[N_v^{\,2}\right] = \mathbf{E}\left[N_f^{\,2}\Delta t^2\right] = Q_f \Delta t^2 \tag{43}$$

where Q_f is the noise covariance of acceleration. Similarly, the noise covariance of orientation that is single integration of gyroscope output is:

$$Q_{angle} = \mathbf{E}\left[N_{angle}^{\,2}\right] = \mathbf{E}\left[N_{gyro}^{\,2}\Delta t^2\right] = Q_{gyro}\Delta t^2 \tag{44}$$

where N_{gyro} is a white noise of gyroscope and Q_{gyro} is the noise covariance gyroscope. The process noise covariance is:

$$\mathbf{Q} = \begin{bmatrix} Q_{\Delta v_N} & 0 & 0 & 0 & 0 & 0 \\ 0 & Q_{\Delta v_E} & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{\Delta v_\phi} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{\Delta v_\theta} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{\Delta v_\varphi} & Q_{\Delta v_\varphi} \\ 0 & 0 & 0 & 0 & Q_{\Delta v_\varphi} & Q_{\Delta v_\varphi} \end{bmatrix} \tag{45}$$

where $Q_{\Delta v_N} = Q_{f_N}\Delta t^2$, $Q_{\Delta v_E} = Q_{f_E}\Delta t^2$, $Q_{\Delta v_\phi} = Q_{g_x}\Delta t^2$, $Q_{\Delta v_\theta} = Q_{g_y}\Delta t^2$, $Q_{\Delta v_\varphi} = Q_{g_z}\Delta t^2$ and $\left[Q_{f_N} \quad Q_{f_E} \quad Q_{f_D}\right]^T = \mathbf{C}_b^n .* \mathbf{C}_b^n \left[Q_{f_x} \quad Q_{f_y} \quad Q_{f_z}\right]^T$. Q_{f_x}, Q_{f_y}, Q_{f_z} are the error covariance measured with the tri-axis accelerometers. Q_{g_x}, Q_{g_y}, Q_{g_z} are the error covariance measured with the tri-axis gyroscopes. Operator '.*' is the array multiplier in MATLAB.

The measurement noise covariance is the error covariance that is measured with the encoders and the compass sensor and it is the error covariance of the attitude calculated by the accelerometers. In this case, we assume that the correlation between the attitude from accelerometers, the output of compass sensor, and the outputs of the encoders are zero. The measurement noise covariance is therefore:

$$\mathbf{R} = \begin{bmatrix} R_{v_N} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{v_E} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & R_\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & R_\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{\Delta\varphi} \end{bmatrix} \tag{46}$$

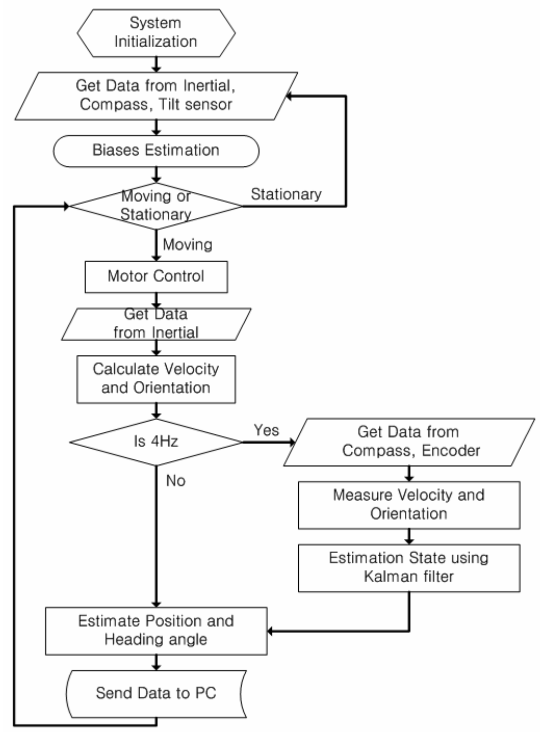


Fig. 5. The flowchart of the proposed system.

where R_{v_N}, R_{v_E} and $R_{\Delta\varphi}$ are the error covariance of the velocity and yaw angle rates estimated by the encoders. R_ϕ and R_θ are the error covariance of the attitude calculated by the accelerometer. R_φ is the error covariance of the yaw angle measured with the compass sensor. Fig. 5 depicts the flowchart of the proposed system.

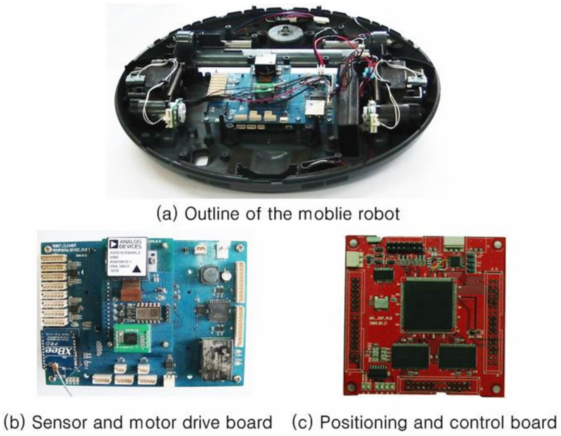
4. The experiment

4.1 The system configuration

The mobile robot is propelled by two DC motors. Each motor was equipped with two channel encoder that measures the rotating velocity and rotating direction of the wheels. An ADIS16354, Analog Device Inc., inertial sensor module was used to measure the acceleration and angular rate. In order to measure the absolute yaw angle, we used a Honeywell HMC6352. The tilt sensor used to compensate for the accelerometer bias is an SCA100T that uses a dual-axial inclinometer made by VTI technologies. A TMS320F28335 DSP, Texas Instrument Inc., was used to calculate the proposed dead reckoning algorithm and to control the mobile robot. Fig. 6 shows (a) the mobile robot, (b) the designed sensor and motor drive board and (c) the designed positioning and control board.

4.2 The bias compensation experiment

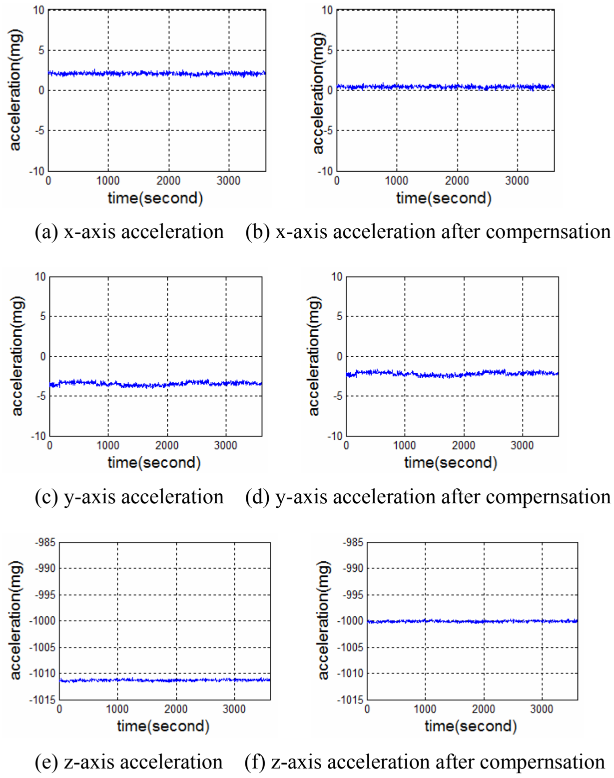
In order to perform the stationary accelerometer bias compensation, we collected 200 samples from the tilt and compass sensors for 10 seconds and then averaged collected samples.



(a) Outline of the moblie robot

(b) Sensor and motor drive board (c) Positioning and control board

Fig. 6. The implemented system.



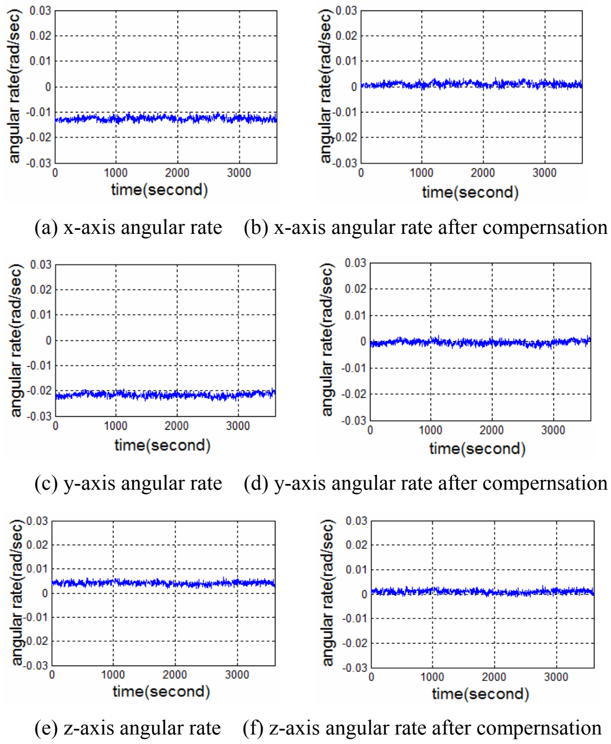
(a) x-axis acceleration (b) x-axis acceleration after compensation

(c) y-axis acceleration (d) y-axis acceleration after compensation

(e) z-axis acceleration (f) z-axis acceleration after compensation

Fig. 7. The acceleration on the body frame.

The transformation matrix was calculated using these outputs (roll, pitch and yaw) and was applied to proposed algorithm. Similarly, the bias values for the gyroscope were calculated using the average value of 200 collected samples. Fig. 7 and Fig. 8 show the outputs of the tri-axis accelerometers and the tri-axis gyroscopes for 3600 seconds after the bias compensation using the proposed algorithm. In Fig. 7(d), at the y-axis is shown in regards to the ground inclination. In Fig. 8, the angular rates are almost zero after the bias compensation. Fig. 9 depicts the orientation of the mobile robot determined by the initial alignment algorithm using the tri-accelerometers and the compass sensor. In Fig. 9(a), the roll angle is shown in regards to the ground inclination. In Fig. 9, the accelerations on navigation frame are almost zero because we are compensated for the ground inclination by using the tilt and compass sensors.

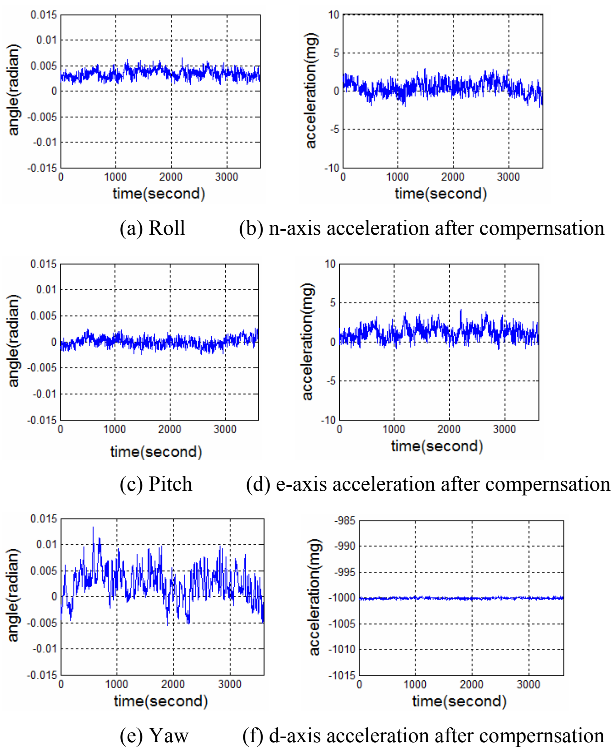


(a) x-axis angular rate (b) x-axis angular rate after compernsation

(c) y-axis angular rate (d) y-axis angular rate after compernsation

(e) z-axis angular rate (f) z-axis angular rate after compernsation

Fig. 8. The angular rate on the body frame.

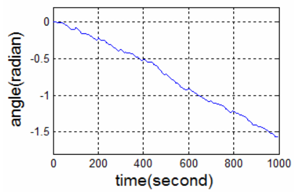


(a) Roll (b) n-axis acceleration after compernsation

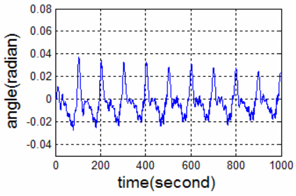
(c) Pitch (d) e-axis acceleration after compernsation

(e) Yaw (f) d-axis acceleration after compernsation

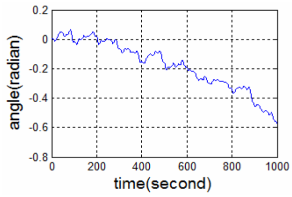
Fig. 9. The orientation and the acceleration on the body frame.



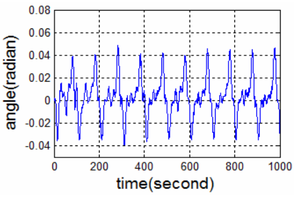
(a) Roll from gyro



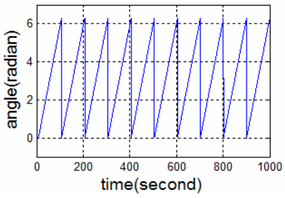
(b) Roll from Kalman



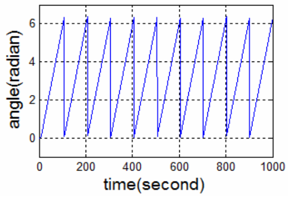
(c) Pitch from gyro



(d) Pitch from Kalman

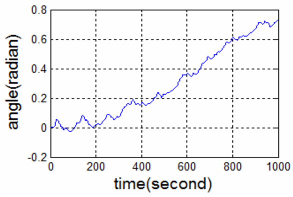


(e) Yaw from gyro

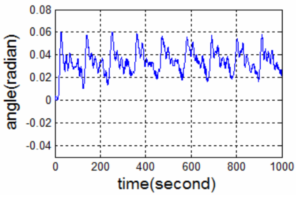


(f) Yaw from Kalman

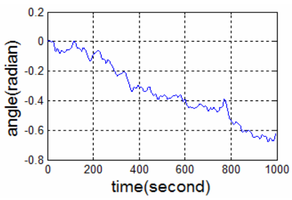
Fig. 10. The CW orientation.



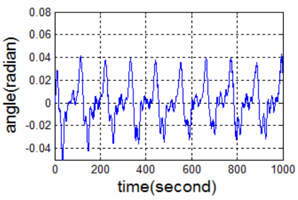
(a) Roll from gyro



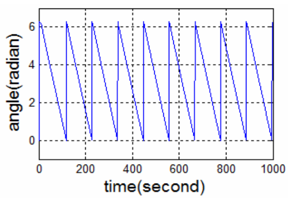
(b) Roll from Kalman



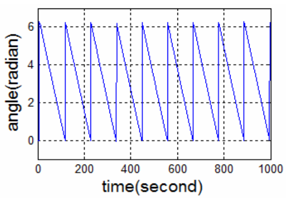
(c) Pitch from gyro



(d) Pitch from Kalman



(e) Yaw from gyro

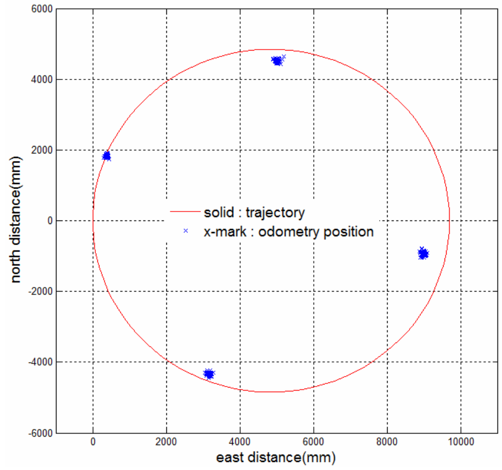


(f) Yaw from Kalman

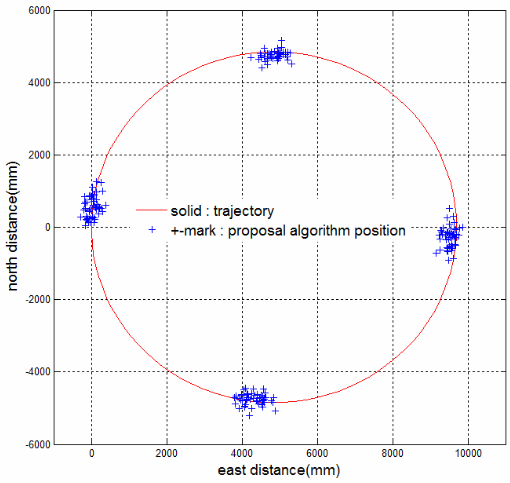
Fig. 11. The CCW orientation.

4.3 The orientation estimation experiment

In order to perform the orientation estimation, the mobile robot was programmed to travel in a 9700 mm diameter circle in the test space. Figs. 10 and 11 depict the orientation of the



(a) Estimation position by odometry



(b) Estimation position by Kalman filter

Fig. 12. The CW estimation position.

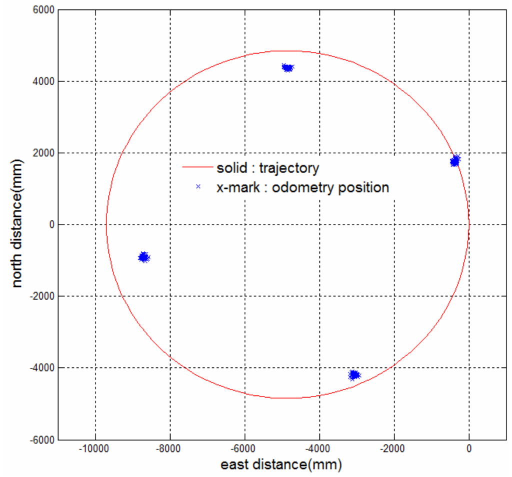
mobile robot in a clockwise (CW) and counterclockwise (CCW) direction, respectively. The pitch angle from the gyroscope continuously decreased because the mobile robot taken accelerated towards the x-axis. The roll angle from the gyroscopes continuously decreased or increased, due to the centrifugal force generated by the circular motion of the mobile robot. From the experiment result, it can be observed that the output of gyroscope is heavily affected by the movement of the mobile robot. Therefore the orientation must be periodically compensated by using external signal, while the mobile robot is moving in long term. From Figs. 10 and 11, the estimated orientation using proposed algorithm has been experimentally shown to compensate for the accumulative error. Also, the roll and pitch from the proposed algorithm are shown to have periodic angles due to the ground inclination.

4.4 The position estimation experiment

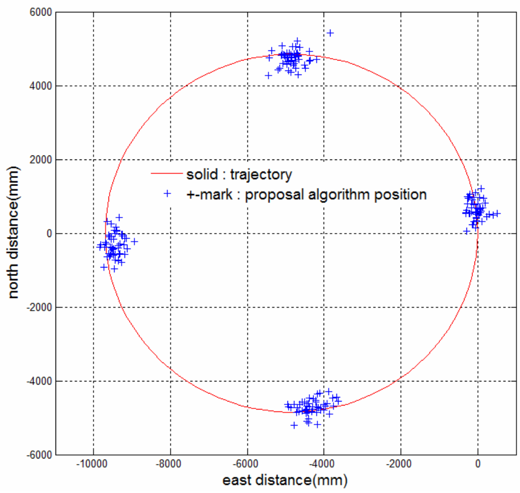
The mobile robot travelled in a 9700 mm diameter circle. The experiments were performed using CW and CCW circu-

Table 1. The error mean and standard deviation.

CW			
Position		Mean	Std. deviation
(4850, 4850)	Odo	(5013, 4534)	(52.37, 36.06)
	Kal	(4946, 4785)	(265.07, 165.51)
(9700, 0)	Odo	(8995, -928.9)	(45.25, 48.69)
	Kal	(9479, -351.2)	(144.14, 368.44)
(4850, -4850)	Odo	(3157, -4345)	(40.54, 37.52)
	Kal	(4258, -4748)	(313.55, 157.02)
(0, 0)	Odo	(368.4, 1826)	(37.81, 50.56)
	Kal	(19.7, 635.8)	(164.53, 283.49)
CCW			
Position		Mean	Std. deviation
(-4850, 4850)	Odo	(-4851, 4371)	(49.87, 37.78)
	Kal	(-4955, 4799)	(250.86, 235.83)
(-9700, 0)	Odo	(-8696, -910.9)	(41.02, 48.05)
	Kal	(-9476, -250.6)	(201.1, 269.84)
(-4850, -4850)	Odo	(-3509, -4190)	(47.03, 42.79)
	Kal	(-4307, -4741)	(376.93, 172.19)
(0, 0)	Odo	(-375.3, 1777)	(39.21, 55.36)
	Kal	(1.54, 619.9)	(173.76, 262.45)



(a) Estimation position by odometry



(b) Estimation position by Kalman filter

Fig. 13. The CCW estimation position.

lar motions, fifty times each. Figs. 12 and 13 depict the positions of the mobile robot. In the figures, the circle is the true position of the mobile robot. In the experiment results, the odometry method estimates a smaller circular position than true position because the calculated yaw angle changes faster than the true yaw angle. Although the result has not been provided in this paper, the positioning using only inertial sensor has shown unbounded position error due to double integration. The proposed method delivers a reliable position with an acceptable estimation error compared to the method that only used odometry or inertial sensors. However, whereas the proposed method provided a small error it had a large variance. Table 1 display the estimation position mean and standard deviation at each of the four positions.

5. Conclusions

An INS offers a quick and accurate response in the short term, but the position estimation errors are amplified due to the integrative nature of the INS, low frequency noise, and sensor bias. The position estimation using encoders will suffer from negative bias due to systematic and nonsystematic errors, and unbounded position errors due to the integrative nature of the rotating speed.

In this paper, we presented mobile robot position estimation that combines odometry and the INS in order to reduce the accumulation errors inherent in dead reckoning. In order to compensate for weaknesses found in each system, we designed a Kalman filter. The proposed system estimates the orientation of the mobile robot using inertial sensors, therefore it can compensate for the inclination of the ground. Inertial sensors have bias errors; the proposed method corrected the bias errors. The proposed method also compensates for the yaw angle errors that generate position errors in odometry therefore it has smaller position errors. Although the proposed system has a comparatively large variance, the system shows a reliable level of position estimation. In the proposed method, the dead reckoning period that estimates the position without any external position data from an absolute positioning system has been extended. In order to improve the position estimation accuracy, the system modeling algorithm and the two systems' combination algorithm need further study.

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References

[1] J. Borenstein and L. Feng, Measurement and correction of systematic odometry errors in mobile robots, *IEEE Trans. Robotics and Automation*, 12 (6) (1996) 869-880.

[2] N. Houshangi and F. Azizi, Accurate mobile robot position determination using unscented Kalman filter, *Electrical and*

Computer Engineering, Canadian Conference (2005) 846-851.
[3] M. Ibraheem, Gyroscope-enhanced dead reckoning localization system for an intelligent walker, *Information Networking and Automation (ICINA), International Conference*, 1 (2010) V1-67~V1-72.
[4] W. S. Moon, B. S. Cho, J. W. Jang and K. R. Baek, A multi-robot positioning system using a multi-code ultrasonic sensor network and a Kalman filter, *International Journal of Control, Automation, and Systems*, 8 (6) (2010) 1349-1355.
[5] A. Widyotriatmo and K. S. Hong, Navigation function-based control of multiple wheeled vehicles, *IEEE Trans. Industrial Electronic*, 58 (5) (2011) 1896-1906.
[6] C. Nakagawa, Y. Suda, K. Nakano and S. Takehara, Stabilization of a bicycle with two-wheel steering and two-wheel driving by driving forces at low speed, *Journal of Mechanical Science and Technology*, 23 (4) (2009) 980-986.
[7] A. Widyotriatmo, B. H. Hong and K. S. Hong, Predictive navigation of an autonomous vehicle with nonholonomic and minimum turning radius constraints, *Journal of Mechanical Science and Technology*, 23 (2) (2009) 381-388.
[8] J. B. Song and K. S. Byun, Steering control algorithm for efficient drive of a mobile robot with steerable omni-directional wheels, *Journal of Mechanical Science and Technology*, 23 (10) (2009) 2747-2756.
[9] D. H. Titterton and J. L. Weston, *Strapdown inertial navigation technology*, Second Ed. The Institute of Electrical Engineers, United Kingdom (2004).
[10] P. G. Savage, Strapdown inertial navigation integration algorithm design part 1: Attitude algorithm, *Journal of Guidance, Control, and Dynamics*, 21 (1) (1998) 19-28.
[11] P. G. Savage, Strapdown inertial navigation integration algorithm design part 2: Velocity and position algorithms, *Journal of Guidance, Control, and Dynamics*, 21 (2) (1998) 208-221.
[12] J. M. Choi, S. J. Lee and M. C. Won, Self-learning navigation algorithm for vision-based mobile robots using machine learning algorithms, *Journal of Mechanical Science and Technology*, 25 (1) (2011) 247-254.
[13] Q. Honghui and J. Moore, Direct Kalman filtering approach for GPS/INS integration, *IEEE Trans. on Aerospace and Electronic Systems,* 38 (2) (2002) 687-693.
[14] C. J. Wu and C. C. Tsai, Localization of an autonomous mobile robot based on ultrasonic sensory information, *Journal of Intelligent and Robotic Systems*, 30 (3) (2001) 267-277.
[15] L. Kleeman, Optimal estimation of position and heading for mobile robots using ultrasonic beacons and dead-reckoning, *Proc. of the 1992 IEEE int. Conf. on Robotics and Automation, Nice, France* (1992) 2582-2587.
[16] C. C. Tsai, A localization system of a mobile robot by fusing dead-reckoning and ultrasonic measurements, *Instrumentation and Measurement Technology Conference, IEEE*, 1 (1998) 144-149.
[17] T. T. Q. Bui and K. S. Hong, Sonar-based obstacle avoidance using region partition scheme *Journal of Mechanical Science and Technology*, 24 (1) (2010) 365-372.
[18] J. Vaganay, M. J. Aldon and A. Fournier, Mobile robot attitude estimation by fusion of inertial data, *Proc. of the* 1993 *IEEE int. Conf. on Robotics and Automation*, 1 (1993) 277-282.
[19] B. Barshan, Hugh F and Durrant-Whyte, Inertial navigation systems for mobile robots, *IEEE Trans. Robotics and Automation,* 11 (3) (1995) 328-342.
[20] G. A. Piedrahita and D. M. Guayacundo, Evaluation of accelerometers as inertial navigation system for mobile robots, *Robotics Symposium, 2006. LARS '06. IEEE 3rd Latin American* (2006) 84-90.
[21] G. Welch and G. Bishop, *An introduction to the Kalman filter*, UNC-Chapel Hill. TR 95-041 (4) (2006).

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