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Multidisciplinary optimization of a stiffened shell by genetic algorithm †

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Abstract

Vibration analysis of simply supported rotating cross-ply laminated stiffened cylindrical shell is performed using an energy approach which includes variational and averaging method. The stiffeners include rings and stringers. The equations are obtained by Rayleigh-Ritz method and Sander's relations. To validate the present method, the results are compared to the results available in other literatures. A good adoption is observed in different type of results including isotropic shells, rotating laminated shells, stiffened isotropic shells and stiffened laminated shells. Then, the optimization of parameters due to shell and stiffeners is conducted by genetic algorithm (GA) method under weight and frequency constraints. Stiffener shape, material properties and dimensions are also optimized.

Keywords: Stiffened shell; Weight; Natural frequency; Genetic algorithm

1. Introduction

Rotating shells and shafts are used in many industrial applications as the main parts of many machines, such as gas turbines, locomotive engines, electric motors, rotor systems and fuel tanks. In many cases, a rotating shell may be one of the main sources of vibration and noise. To reduce the vibration, noise and increasing strength and to enhance the stiffness, the shells and shafts are usually made of laminated composite materials or reinforced by stiffener. The stiffened cylindrical shell with beam type elements is extensively used in mechanical structures, such as aircraft fuselages, commercial vehicles, road tankers, missiles, and submarines. Therefore, it's very important for engineers to understand the vibration of composite shells in order to design suitable shells with low vibration and noise production characteristics. Hence, vibration characteristics of rotating stiffened cylindrical shells are of great importance.

In recent years, as composite materials have advantages of high strength-to-weight ratio, advanced composite materials have been used widely in many fields of engineering. Numerous methods have been developed and used to study the vibration behavior of thin shells. These methods range from energy methods based on the Rayleigh-Ritz procedure to analytical methods in which, respectively, closed-form solutions of the governing equations and iterative solution approaches were used. There are many studies on these rotating structures without stiffeners. Previously published papers, however, are primarily concerned with the stiffened isotropic shell and many studies have been carried out on laminated cylindrical shells with or without the rotation. But papers about optimization of related parameters are rare and they considered some individual parameters. In the present paper, a full scale of all parameters is considered for optimization process and also the analysis is performed for rotating shell with stiffeners made of isotropic and composite materials.

The study of the vibrations for the composite cylindrical shells has been reported by many researchers [1-4]. They have studied the effects of various parameters such as boundary conditions, fiber orientation angles and material properties of the composite shells on the vibration characteristics. But only several researchers have investigated the vibrations of the combined shell with an interior or exterior plate. Egle and Sewall [5] investigated the free vibration for a ring and stringer stiffened cylindrical shell with various boundary conditions. Peterson and Boyd [6] developed an analytical approach by using the Rayleigh-Ritz method to study the free vibration of a circular cylindrical shell partitioned by an interior rectangular plate. Zinberg and Symonds [7] experimentally obtained the critical speed of rotating cylindrical shells. The results also proved the advantages of using shells made of orthotropic materials over aluminum alloy shells. Singer et al. [8] developed a method for the vibration analysis of a preloaded stiffened cylindrical panel with various boundary conditions along

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the straight edges. The smeared stiffener approach was used in the analysis. A finite element approach was used by dos Reis et al. [9] to obtain the critical speeds in the evaluation of the experiments of Zinberg and Symonds [8]. ESDU [10] (Engineering Science Data Unit) have published a computer program for the orthogonally stiffened shells.

Irie et al. [11] studied the free vibration of non-circular cylindrical shells with longitudinal interior partitions by using the transfer matrix. Mustafa and Ali [12] presented a concise yet comprehensive method for the determination of natural frequency of ring, stringer and orthogonally stiffened cylindrical shells based on the formulation of energy. Lee and Kim [13] studied the minimum weight design for the orthogonally stiffened cylindrical shell subjected to axial compression. The first work on the rotating cylindrical shell was conducted by Bryan [14], who studied the free vibration of the rotating cylindrical shell and discovered the phenomenon of travelling modes. Reddy and Starnes [15] presented the analytical solution for buckling loads of composite cylindrical shells with axial and circumferential stiffeners, where stiffeners are isotropic. The buckling and vibration analysis for the stiffened composite cylindrical shells with cutout was investigated by Lee and Hur [16] using FEM code, ANSYS and ABAQUS software.

A simplified theory for analyzing the first critical speed of a composite cylindrical shell was given by Kim and Bert [17] and a comparison with various shell theories was made. Schokker et al. [18] investigated the dynamic instability associated with the interactive buckling of ring stiffened composite shells under hydrostatic pressure. Goswani and Mudhopadhyay [19] presented the geometrically nonlinear elastic transient response analysis of composite stiffened panels using finite element method. For the rotating cylindrical shell, much research has been reported. Using the finite element method, Kim and Lee [20] presented the vibration characteristic of shell of revolution including Coriolis effect. Huang and Soedel [21] investigated the free and forced vibrations of rotating simply supported cylindrical shell. Huang and Soedel [22] presented the results of an analysis of both ends of a simply supported cylindrical shell with a plate at an arbitrary axial position. Suzuki et al. [23] studied the free vibration of rotating thin cylindrical shells using power series expansions. Langley [24] applied a dynamic stiffness technique for the vibration analysis of a simply supported stiffened shell structure. Igawa et al. [25] examined the free vibration of rotating anisotropic shells of revolution including the shear deformation. Lam and Loy [26] presented the vibration characteristics for the GFRP composite laminate cylindrical shell using different shell theories. Missaoui et al. [27] studied the free and forced vibration of a cylindrical shell with a floor partition based on a variational formulation in which the structural coupling is simulated using artificial spring systems. Lee and Kim [28] studied the linear and nonlinear vibration for the rotating hybrid cylindrical shells. The Ritz-Galerkin method is applied to obtain the nonlinear frequency equation. In other papers [29, 30] they mentioned rotating stiffened cylindrical shell by using the energy method and they considered the various boundary conditions for rotating composite cylindrical shells with the orthogonal stiffeners by using the Rayleigh-Ritz procedure based on the energy method. Lam and Li [31] investigated rotating conical orthotropic shells based on Love's first approximation theory. Yim et al. [32] applied a new method to analyze the free vibration of clamped-free shell with a plate attached at an arbitrary axial position.

A free vibration analysis of rotating cylindrical shells, using the layerwise laminate theory and only for simply supported boundary conditions, was performed by Kadivar and Samani [33]. The free vibration analysis of a rotating isotropic cylindrical shell using a harmonic reproducing kernel particle method was performed by Liew et al. [34]. The vibrations of rotating cross-ply and general laminated composite cylindrical shells based on Love's equations of motion and using a wave propagation approach have been investigated by Zhang [35, 36]. Lee et al. [37] analyzed the free vibration of simply supported composite cylindrical combined shells with an interior rectangular plate by using Rayleigh-Ritz procedure based on the energy principle. Nayak and Bandyopadhyay [38] used finite element analysis for free vibration behavior of doubly curved stiffened shallow shells. In another paper [39], they performed the free vibration of the laminated composite anticlastic doubly curved stiffened shells by using the finite element method. Zhao et al. [40] performed vibration analysis of rotating cross-ply laminated circular cylindrical shells with stringer and ring stiffeners by using Love's relations. Reddy's layerwise theory is combined with a wave propagation approach by Ramezani and Ahmadian [41] to study all the conventional boundary conditions in our analysis by using Hamilton's principle.

Papers about optimization of vibration characteristics of cylindrical shells are rare. Mesquita and Kamat [42] considered the maximization of frequencies of stiffened laminated composite plates subject to frequency separation constraints and an upper bound on weight. Sun and Mao [43] presented the buckling of stiffened laminated composite circular cylindrical shells subjected to axial compression or hydrostatic pressure. Taking the fiber orientations of lamina as optimizing parameters, a two-step optimization procedure was employed to maximize the buckling load of stiffened composite cylinders. Seibel et al. [44] investigated thin-walled, unstiffened and stiffened shell structures made of fiber composite materials which are frequently applied due to their high stiffness/strength to weight ratios. Marcelin [45] gave the application of stochastic techniques for the optimization of stiffened plates or shells. Two examples were presented, the first one deals with the optimization of stiffeners on a plate by varying their positions, while having well-defined dimensions; the second example deals with the optimization of a thin shell subject to buckling. The optimum design of stiffened shell structures is investigated using a robust and efficient optimization algorithm performed by Lagaros et al. [46] where the total

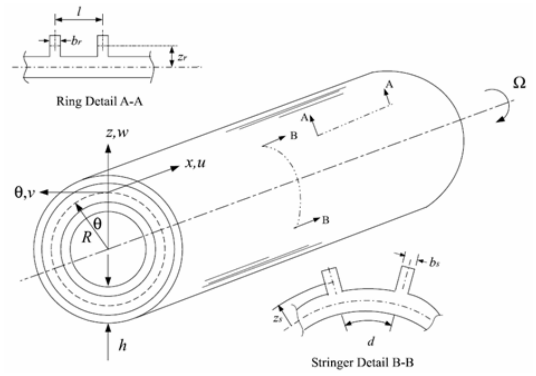

Fig. 1. Coordinate system and stiffener and ring cross section area for the rotating stiffened cylindrical shell.

weight of the structure was to be minimized subject to behavioral constraints imposed by structural design codes. Kegl and Brank [47] presented an effective approach to shape optimal design of statically loaded elastic shell-like structures. The shape parameterization was based on a design element technique. Jarmai [48] presented the multi-objective optimization of a welded stringer-stiffened cylindrical steel shell. Variables are the shell thickness as well as dimension and number of stringers.

In this study, genetic algorithm (GA) optimization of rotating isotropic and laminated shells is conducted under weight and frequency constraints, considering a full scale of all parameters. Thus, vibration analysis of simply supported rotating cross-ply laminated stiffened cylindrical shell is performed using an energy approach for different types of parameters.

2. Theoretical formulations

2.1 General approach

However, the analysis of vibration characteristics of cylindrical shells is more complex than that of beams and plates, mainly because the motion equations of cylindrical shells together with boundary conditions are more complex. Love [49] modified the Kirchhoff hypothesis for plates and established the preconditions of the so-called classic theory of thin shells, which is now commonly known as Love's first approximation of the first kind. He then subsequently formulated a shell theory known as Love's first approximation theory and the preconditions he established soon became the foundations on which many thin shell theories were later developed such as Flugge theory [50].

The stiffened cylindrical shell, as shown in Fig. 1, is considered to be thin, laminated and composed of an arbitrary number layers with parameters length L, radius R, thickness h, and is rotating about the x-axis at constant angular velocity, Ω. A coordinate system (x,θ,z) is fixed on the middle surface of the shell. The displacements of the shell in the x,θ,z directions are denoted by u, v, and w respectively. The depths of the stringer and ring are denoted by d_s and d_r, and the corresponding widths by b_s and b_r, respectively. The displacements from the middle surface of the shell to the centroid of the stringer and ring are denoted by Z_s and Z_r, respectively.

The strain vector can be written as

$$\{\varepsilon\}^T = \{e_1 \quad e_2 \quad e_3 \quad k_1 \quad k_2 \quad k_3\} \tag{1}$$

where the middle surface strains, e_1, e_2, e_3 and the middle surface curvatures, k_1, k_2, k_3 are defined according to Sander's theory as follows:

$$e_1 = \frac{\partial u}{\partial x},\ e_2 = \frac{1}{R}\left(w + \frac{\partial v}{\partial \theta}\right),\ e_3 = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}$$
$$k_1 = -\frac{\partial^2 w}{\partial x^2},\ k_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right)$$
$$k_3 = \frac{1}{R}\left(\frac{\partial v}{\partial x} - 2\frac{\partial^2 w}{\partial x \partial \theta}\right). \tag{2}$$

For stringers, the displacements in the x,θ,z directions are defined as

$$u_s = u - z\frac{\partial w}{\partial x},\ v_s = v - \frac{z}{R}\frac{\partial w}{\partial \theta},\ w_s = w. \tag{3}$$

The strain of stringers in the axial direction is described as

$$\varepsilon_s = \frac{\partial u_s}{\partial x}. \tag{4}$$

For rings, the displacements in the x,θ,z directions are defined as

$$u_r = u - z_r\frac{\partial w}{\partial x},\ v_r = v - \frac{z_r}{R}\frac{\partial w}{\partial \theta},\ w_r = w. \tag{5}$$

The strain of rings in the radial direction is described as

$$\varepsilon_r = \frac{1}{R}\left(w_r + \frac{\partial v_r}{\partial \theta}\right). \tag{6}$$

If the shell is assumed to be simply supported, the displacement components can be approximated in the term of time (t) as

$$u = A\cos\left(\frac{m\pi x}{L}\right)\cos(n\theta + \omega t)$$
$$v = B\sin\left(\frac{m\pi x}{L}\right)\sin(n\theta + \omega t) \tag{7}$$
$$w = C\sin\left(\frac{m\pi x}{L}\right)\cos(n\theta + \omega t)$$

where m represents the number of axial half wave, n represents the number of circumferential half wave and ω is the natural frequency of the rotating shell. Rayleigh-Ritz method is an approach by considering some functions with unknown coefficients in Eq. (7) for displacements and then by replacing them in the governing equation and differentiating from unknown coefficients, natural frequencies of system can be obtained.

2.2 Strain energy of shell

The strain energy of the shell is expressed as

$$U_e = \frac{1}{2}\int_0^L\int_0^{2\pi} \{\varepsilon\}^T [S] \{\varepsilon\} R d\theta dx . \quad (8)$$

It should be mentioned that h is not in the strain energy of the shell because of the existence of h effect in [S], the stiffness matrix which can be written as

$$[S]=\begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \quad (9)$$

where A_{ij}, B_{ij} and D_{ij} are defined as extensional, coupling and bending stiffness respectively. For a shell composed of different layers of orthotropic material, these stiffnesses can be written as

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{N} \tilde{Q}_{ij}^{(k)} \left(h_k - h_{k+1}\right) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{N} \tilde{Q}_{ij}^{(k)} \left(h_k^2 - h_{k+1}^2\right) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{N} \tilde{Q}_{ij}^{(k)} \left(h_k^3 - h_{k+1}^3\right) \end{aligned} \quad (10)$$

where h_k and h_{k+1} denote the distance from the shell reference surface (middle surface) to the outer and inner surfaces of the k-th layer. Then, N is the number of layers in the laminated shell and $\tilde{Q}_{ij}^{(k)}$ is the transformed reduced stiffness matrix for the k-th layer which is defined as

$$[\tilde{Q}] = [T][Q][T]^{-1} \quad (11)$$

where [T] is the transformation matrix for the principal material coordinates and the shell coordinates system and is defined as

$$[T]=\begin{bmatrix} \cos^2\alpha & \sin^2\alpha & \sin 2\alpha \\ \sin^2\alpha & \cos^2\alpha & -\sin 2\alpha \\ -\frac{1}{2}\sin 2\alpha & \frac{1}{2}\sin 2\alpha & \cos 2\alpha \end{bmatrix} \quad (12)$$

where α is orientation of the fibers and [Q] is the reduced stiffness matrix which is defined as

$$[Q]=\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}. \quad (13)$$

And the material constants in the reduced stiffness matrix are given as

$$\begin{aligned} Q_{11} &= \frac{E_{11}}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_{22}}{1-\nu_{12}\nu_{21}}, \quad Q_{11} = G_{12} \end{aligned} \quad (14)$$

where E_{11} and E_{22} are the elastic modulus, G_{12} is the shear modulus and ν_{12} and ν_{21} are the Poisson ratios. The strain energy of shell due to hoop tension is

$$\begin{aligned} U_{h,e} &= \frac{h}{2}\int_0^L\int_0^{2\pi} N_{\theta,e}\vec{U}_{h,e} R d\theta dx \\ \vec{U}_{h,e} &= \left[\frac{1}{R}\frac{\partial u}{\partial\theta}\right]^2 + \left[\frac{1}{R}\left(w+\frac{\partial v}{\partial\theta}\right)\right]^2 + \left[\frac{1}{R}\left(v-\frac{\partial w}{\partial\theta}\right)\right]^2 \end{aligned} \quad (15)$$

where the initial hoop tension due to centrifugal force is defined as

$$N_{\theta,e} = \rho R^2 \Omega^2 \quad (16)$$

where ρ is the shell density. The kinetic energy of the rotating shell is given by

$$\begin{aligned} T_e &= \frac{\rho h}{2}\int_0^L\int_0^{2\pi} \vec{T}_e R d\theta dx \\ \vec{T}_e &= \dot{u}^2 + \dot{v}^2 + \dot{w}^2 + 2\Omega(v\dot{w} - \dot{v}w) + \Omega^2(v^2 + w^2) \end{aligned} \quad (17)$$

where $\dot{u},\dot{v},\dot{w}$ are the components of the velocity in the x,θ,z direction, respectively.

2.3 Strain energy of rings

The strain energy of the rings, using variational method is

$$U_r = \frac{1}{2}\sum_{k=1}^{N_r}\int_0^{2\pi}\vec{U}_r R d\theta$$

$$\vec{U}_r = \int_{A_{r,k}} E_{r,k}\varepsilon_r^2 dA_{r,k} + G_{r,k}J_{r,k}\left(\frac{1}{R}\frac{\partial^2 w_r}{\partial x\partial\theta}\right)^2 \quad (18)$$

where N_r is the number of rings, $E_{r,k}$, $A_{r,k}$ and $G_{r,k}J_{r,k}$ are the elastic modulus, cross sectional area and torsional stiffness of the k-th ring, respectively. The strain energy of the rings, by using averaging method is expressed as

$$\tilde{U}_r = \frac{1}{2l}\int_0^L\int_0^{2\pi}\left\{\int_{A_r} E_r\varepsilon_r^2 dA_r + G_r J_r\left(\frac{1}{R}\frac{\partial^2 w_r}{\partial x\partial\theta}\right)^2\right\}Rd\theta dx \quad (19)$$

where l is the distance between rings. In averaging method, integrals are used instead of summation (in variational method) due to reduce the time duration of calculations. In other words, it's an approximate method in comparison with variational method. The strain energy of the rings due to hoop tension, by using variational method is taken to be

$$U_{h,r} = \frac{1}{2}\sum_{k=1}^{N_r}\int_0^{2\pi}\int_{A_{r,k}} N_{\theta,r}\vec{U}_{h,r} dA_{r,k} R d\theta$$

$$\vec{U}_{h,r} = \left[\frac{1}{R}\frac{\partial u_r}{\partial\theta}\right]^2 + \left[\frac{1}{R}\left(w_r + \frac{\partial v_r}{\partial\theta}\right)\right]^2 + \left[\frac{1}{R}\left(v_r - \frac{\partial w_r}{\partial\theta}\right)\right]^2 \quad (20)$$

where the initial hoop tension of rings due to centrifugal force is defined as

$$N_{\theta,r} = \rho_r R^2\Omega^2 \quad (21)$$

where ρ_r is the density of the k-th ring. The strain energy of the rings due to hoop tension, by using averaging method is taken to be

$$\tilde{U}_{h,r} = \frac{1}{2l}\int_0^L\int_0^{2\pi}\int_{A_r} N_{\theta,r}\vec{\tilde{U}}_{h,r} dA_r R d\theta dx$$

$$\vec{\tilde{U}}_{h,r} = \left[\frac{1}{R}\frac{\partial u_r}{\partial\theta}\right]^2 + \left[\frac{1}{R}\left(w_r + \frac{\partial v_r}{\partial\theta}\right)\right]^2 + \left[\frac{1}{R}\left(v_r - \frac{\partial w_r}{\partial\theta}\right)\right]^2. \quad (22)$$

The kinetic energy of rings, using variational method is

$$T_r = \frac{\rho_r}{2}\sum_{k=1}^{N_r}\int_0^{2\pi}\int_{A_{r,k}}\vec{T}_r dA_{r,k} R d\theta$$

$$\vec{T}_r = \dot{u}_r^2 + \dot{v}_r^2 + \dot{w}_r^2 + 2\Omega(v_r\dot{w}_r - \dot{v}_r w_r) + \Omega^2(v_r^2 + w_r^2). \quad (23)$$

The kinetic energy of rings, by using averaging method is given by

$$\tilde{T}_r = \frac{\rho_r}{2l}\int_0^L\int_0^{2\pi}\int_{A_r}\vec{\tilde{T}}_r dA_r R d\theta dx$$

$$\vec{\tilde{T}}_r = \dot{u}_r^2 + \dot{v}_r^2 + \dot{w}_r^2 + 2\Omega(v_r\dot{w}_r - \dot{v}_r w_r) + \Omega^2(v_r^2 + w_r^2). \quad (24)$$

2.4 Strain energy of stringers

The strain energy of the stringers, by using variational method is expressed as

$$U_s = \frac{1}{2}\sum_{k=1}^{N_s}\int_0^L\left\{\int_{A_{s,k}} E_{s,k}\varepsilon_s^2 dA_{s,k} + G_{s,k}J_{s,k}\left(\frac{1}{R}\frac{\partial^2 w_s}{\partial x\partial\theta}\right)^2\right\}dx \quad (25)$$

where N_s is the number of stringers, $E_{s,k}$, $A_{s,k}$ and $G_{s,k}J_{s,k}$ are the elastic modulus, cross sectional area and torsional stiffness of the k-th stringer, respectively. The strain energy of the stringers, by using averaging method is expressed as

$$\tilde{U}_s = \frac{1}{2d}\int_0^L\int_0^{2\pi}\left\{\int_{A_s} E_s\varepsilon_s^2 dA_s + G_s J_s\left(\frac{1}{R}\frac{\partial^2 w_s}{\partial x\partial\theta}\right)^2\right\}Rd\theta dx \quad (26)$$

where d is the distance between stringers. The strain energy of the stringers due to hoop tension is zero. The kinetic energy of stringers, by using variational method is given by

$$T_s = \frac{\rho_s}{2}\sum_{k=1}^{N_s}\int_0^L\int_{A_{s,k}}\vec{T}_s dA_{s,k} dx$$

$$\vec{T}_s = \dot{u}_s^2 + \dot{v}_s^2 + \dot{w}_s^2 + 2\Omega(v_s\dot{w}_s - \dot{v}_s w_s) + \Omega^2(v_s^2 + w_s^2) \quad (27)$$

where ρ_s is the density of the k-th stringer. The kinetic energy of stringers, by using averaging method is given by

$$\tilde{T}_s = \frac{\rho_s}{2d}\int_0^L\int_0^{2\pi}\int_{A_s}\vec{\tilde{T}}_s dA_s R d\theta dx$$

$$\vec{\tilde{T}}_s = \dot{u}_s^2 + \dot{v}_s^2 + \dot{w}_s^2 + 2\Omega(v_s\dot{w}_s - \dot{v}_s w_s) + \Omega^2(v_s^2 + w_s^2). \quad (28)$$

2.5 Total strain energy

Total strain energy includes the energy functional of the shell, rings and stringers, by using variational method can thus be written as

$$\Pi = T_e + T_r + T_s - U_e - U_{h,e} - U_r - U_{h,r} - U_s \quad (29)$$

or by using averaging method

$$\tilde{\Pi} = T_e + \tilde{T}_r + \tilde{T}_s - U_e - U_{h,e} - \tilde{U}_r - \tilde{U}_{h,r} - \tilde{U}_s . \quad (30)$$

Applying variational principles (Rayleigh-Ritz method), by using variational method results as

$$\delta \int_{t_0}^{t_0 + 2\pi/\omega} \Pi dt = 0. \quad (31)$$

The following matrix relationship can be established as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}. \quad (32)$$

For non-trivial solution of Eq. (32), it should be

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 . \quad (33)$$

Expanding Eq. (33), the characteristic frequency equation can be obtained as

$$\beta_6 \omega_{mn}^6 + \beta_5 \omega_{mn}^5 + \beta_4 \omega_{mn}^4 + \beta_3 \omega_{mn}^3 + \beta_2 \omega_{mn}^2 + \beta_1 \omega_{mn} + \beta_0 = 0. \quad (34)$$

For non-rotating shells, the coefficient for odd powers of ω_{mn} do not appear and the coefficient for the term ω_{mn}^5 will be zero for unstiffened rotating shells. Applying variational principles (Rayleigh-Ritz method), by using averaging method results as

$$\delta \int_{t_0}^{t_0 + 2\pi/\omega} \tilde{\Pi} dt = 0. \quad (35)$$

The following matrix relationship can be established as

$$\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}. \quad (36)$$

For non-trivial solution of Eq. (36), it should be

$$\begin{vmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{vmatrix} = 0 . \quad (37)$$

Expanding Eq. (37), the characteristic frequency equation can be obtained as

$$\tilde{\beta}_6 \omega_{mn}^6 + \tilde{\beta}_5 \omega_{mn}^5 + \tilde{\beta}_4 \omega_{mn}^4 + \tilde{\beta}_3 \omega_{mn}^3 + \tilde{\beta}_2 \omega_{mn}^2 + \tilde{\beta}_1 \omega_{mn} + \tilde{\beta}_0 = 0 . \quad (38)$$

3. Results and discussion

A code is written in MATLAB software to solve equations and to check the validity of the present analysis and the code results including the natural frequencies are compared with different literatures listed in Tables 1-4. Table 1 includes the natural frequencies of an isotropic cylindrical shell (non-rotating and unstiffened), Table 2 includes non-dimensional natural frequencies of a [0/90/0] laminated cylindrical shell, Table 3 includes the natural frequencies of an isotropic cylindrical shell (non-rotating) with 20 stringers and 13 rings and Table 4 includes the natural frequencies of a non-rotating [0/90/0] laminated cylindrical shell with stiffeners. All the results show an adoption of the present method to the other references.

4. Optimization

4.1 General approach

To optimize the stiffened cylindrical shells under the weight and the frequency constraints, genetic algorithm (GA) method is used. This is conducted in MATLAB software (optimization tool) in two parts including multidisciplinary optimization of isotropic and laminated rotating cylindrical shell by considering different variables such as thickness, radius and length of the shell, the number of rings and stringers, type of laminated layers, shape type of stiffeners, material type etc. According to the optimization tool of MATLAB software for genetic algorithm, population type and is the double vector with size of 20, for creation function, it uses constraint dependent, fitness scaling function is rank, and selection function is stochastic uniform. In reproduction, elite count is 2 and crossover fraction is 0.8 with a scattered function and for mutation function, constraint dependent is used. Direction of migration is defined as forward with fraction of 0.2 and interval of 20. To optimize the parameters, a ratio is defined ($F_{m,n}$) as below which is the natural frequencies of stiffened shell ($f_{m,n}$) to the weight of stiffened (W_{tot}) shell ratio.

$$F_{m,n} = \frac{f_{m.n}}{W_{tot}} \quad (39)$$

This ratio is a target function which should be maximized. It means that the weight is minimized and natural frequencies are maximized. The constraints are defined as two ratios including one minus weight of stiffened (W_{tot}) to unstiffened (W_0) shell ratio and also the natural frequencies of stiffened ($f_{m.n}$) to unstiffened (f_0) shell ratio minus one.

$$\left(1 - \frac{W_{tot}}{W_0}\right) \ge 0, \ \left(\frac{f_{m,n}}{f_0} - 1\right) \ge 0 \quad (40)$$

Table 1. Natural frequencies (Hz) of an isotropic cylindrical shell (non-rotating), h=0.02 (in.), L=11.74 (in.), R=5.836 (in.), density=0.000734 (lb^2/in^4), E=29500000 (lb/in^2), v=0.285.

N	Natural frequencies (Hz)					
	m=1			m=2		
	Bert et al. [1] (Love's Eq.)	Rath & Das [2] (SDST Eq.)	Present (Sander's Eq.)	Bert et al. [1] (Love's Eq.)	Rath & Das [2] (SDST Eq.)	Present (Sander's Eq.)
1	3271.0	3270.53	3270.51	4837.9	4837.67	4837.37
2	1862.3	1861.95	1861.76	3725.5	3724.98	3724.41
3	1102.0	1101.75	1100.37	2743.7	2742.61	2740.93
4	705.9	706.66	699.27	2018.5	2018.02	2013.49
5	497.9	497.47	475.65	1515.4	1514.96	1503.51

Table 2. Non-dimensional natural frequencies ($\tilde{\omega} = \omega R\sqrt{\rho / E_{22}}$) of a [0/90/0] laminated cylindrical shell, L/R=1, h/R=1; h=0.002 (m), E_{11}=19 (GPa), E_{22}=7.6 (GPa), G_{12}=4.1(GPa), v_{12}=0.26, density=1643 (kg/m^3).

Angular velocity (rev/s)	n	Natural frequencies (rad/s) and non-dimensional natural frequencies, m=1							
		Lam & Loy [26]		Lee & Kim [29]		Zhao et al. [40]		Present	
		Backward	Forward	Backward	Forward	Backward	Forward	Backward	Forward
0.1	1	1.061429	1.061140	-	-	1.061428	1.061139	1.061429	1.061140
0.1	2	0.824214	0.803894	-	-	0.804212	0.803892	0.804214	0.803894
0.1	3	0.598476	0.598157	-	-	0.598472	0.598183	0.598476	0.598187
0.1	4	0.450270	0.450021	-	-	0.450263	0.450015	0.450270	0.450021
0.1	5	0.345363	0.345149	-	-	0.345355	0.345140	0.345363	0.345149
0.1	6	0.270852	0.270667	-	-	0.270840	0.270654	0.270851	0.270667
0.4	1	1.061862	1.060706	1.061850	1.060693	1.061862	1.060705	1.061862	1.060706
0.4	2	0.804696	0.803415	0.804691	0.803410	0.804694	0.803413	0.804696	0.803415
0.4	3	0.598915	0.597762	0.598912	0.597759	0.598911	0.597758	0.598915	0.597762
0.4	4	0.450662	0.449667	0.450658	0.449664	0.450654	0.449660	0.450661	0.449666
0.4	5	0.345724	0.344870	0.345719	0.344866	0.345714	0.344860	0.345723	0.344869
0.4	6	0.271207	0.270468	0.271200	0.270461	0.271193	0.270454	0.271205	0.270466
1.0	1	1.062728	1.059836	1.062716	1.059825	1.062728	1.059837	1.062729	1.059837
1.0	2	0.805667	0.802464	0.805660	0.802457	0.805664	0.802461	0.805666	0.802463
1.0	3	0.599820	0.596937	0.599814	0.596931	0.599813	0.596930	0.599817	0.596934
1.0	4	0.451513	0.449027	0.451506	0.449019	0.451502	0.449015	0.451508	0.449022
1.0	5	0.346593	0.344459	0.346583	0.344448	0.346577	0.344442	0.346586	0.344451
1.0	6	0.272197	0.270349	0.272182	0.270334	0.272174	0.270326	0.272186	0.270339

Table 3. Natural frequencies (Hz) of an isotropic shell (non-rotating), h=0.00204 (m), R=0.203(m), L=0.813 (m), E=207 (GPa), v=0.3, density= 7430 (kg/m^3), with 20 Stringers (0.004 X 0.006) and 13 Rings (0.006 X 0.008).

n	Natural frequencies (Hz), m=1				
	ESDU [10]	Mustafa & Ali [12]	Lee & Kim [29]	Present (averaging)	Present (variational)
1	938	942	947	1037.8	1086.7
2	443	439	458	472.4	523.3
3	348	337	355	353.6	389.3
4	492	482	507	510.7	531.4
5	745	740	776	789.3	804.2

Table 4. Natural frequencies of a non-rotating [0/90/0] laminated cylindrical shell, L/R=4, h/R=0.005; h=0.001 (m), E_{11}=19 (GPa), E_{22}=7.6 (GPa), G_{12}=4.1(GPa), v_{12}=0.26, density=1643 (kg/m^3), b_r=b_s=0.002 (m), d_r=d_s=0.008 (m), E_r=E_s=3E_{11},v=0.3.

n	Natural frequencies (Hz), m=1							
	Zhao et al. [40]		Present		Zhao et al. [40]		Present	
	Stringers/rings: 4/4		Stringers/rings: 4/4		Stringers/rings: 10/5		Stringers/rings: 10/5	
	Variational	Averaging	Variational	Averaging	Variational	Averaging	Variational	Averaging
1	561.4	553.7	571.0	651.4	549.8	549.7	584.4	653.5
2	286.4	288.4	291.9	292.3	299.9	299.8	314.3	298.1
3	261.4	291.6	261.4	234.7	305.1	305.2	287.4	246.9
4	404.5	470.6	395.1	259.7	486.6	486.7	428.8	382.0
5	627.5	734.6	606.9	561.8	756.8	756.8	654.0	596.6
6	906.9	1062.8	872.9	814.4	1093.7	1093.8	937.7	864.4
7	1238.4	1451.5	1188.6	1113.3	1493.0	1493.0	1274.9	1181.2
8	1620.9	1899.7	1552.9	1457.7	1953.3	1953.3	1664.0	1546.2
9	2053.9	2406.9	1965.5	1847.4	2474.1	2474.1	2104.7	1959.0
10	2537.4	2972.8	2426.2	2282.1	3054.9	3054.9	2596.6	2419.4

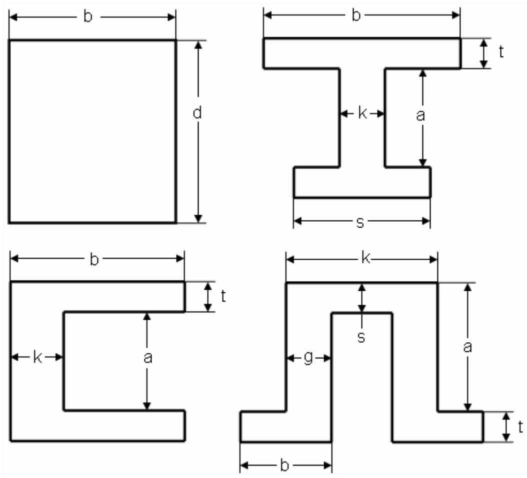

Fig. 2. Different shapes of stiffeners and their defined dimensions.

Thus, the weight of a stiffened shell is minimized in comparison with unstiffened one and the natural frequencies of stiffened shell are maximized in comparison with unstiffened one. By using this target function and constraints, the stiffened shell can be improved in comparison with unstiffened one by the lowest weight and the highest natural frequencies.

4.2 Optimization of stiffener shape

Four shapes (shown in Fig. 2) are considered for stiffeners (both rings and stringers) in an isotropic rotating shell including rectangular shape, C-shape, I-shape and Ω-shape. The details of these shapes are listed in Table 5. The other shell parameters such as material properties of shell and its stiffeners (both 10 rings and 10 stringers) are constants including density defined as 7800 (Kg/m^3), Poisson ratio defined as 0.33 and Young modulus defined as 201.04 (GPa). The shell dimensions, including thickness, radius and length, are considered as 0.1 (m), 0.05 (m) and 0.5 (m), respectively. And also the angular velocity is constant as 0.1 (rev/s).

Table 5. Definition of constants in shape optimization of isotropic rotating shell.

Shape types	Constants	Dimension	Contents
Rectangular	b_r=b_s	m	0.002
	d_r=d_s	m	0.010
	A_r=A_s	m^2	2.000 X 10^{-5}
	J_r=J_s	m^4	6.667 X 10^{-10}
C-Shape	b_r=b_s	m	0.004
	t_r=t_s	m	0.001
	k_r=k_s	m	0.004
	a_r=a_s	m	0.003
	A_r=A_s	m^2	2.000 X 10^{-6}
	J_r=J_s	m^4	1.667 X 10^{-10}
I-Shape	b_r=b_s	m	0.008
	t_r=t_s	m	0.001
	k_r=k_s	m	0.002
	a_r=a_s	m	0.004
	s_r=s_s	m	0.004
	A_r=A_s	m^2	2.000 X 10^{-5}
	J_r=J_s	m^4	2.617 X 10^{-10}
Ω -Shape	b_r=b_s	m	0.004
	t_r=t_s	m	0.001
	k_r=k_s	m	0.004
	a_r=a_s	m	0.004
	s_r=s_s	m	0.001
	g_r=g_s	m	0.001
	A_r=A_s	m^2	2.000 X 10^{-5}
	J_r=J_s	m^4	2.617 X 10^{-10}

The results are listed in Table 6. The target of this optimiza-

Table 6. Ratio of natural frequencies to weights for different stiffener shapes of isotropic rotating shell.

Stiffener types	Shape types	Ratio of natural frequencies to weights (Hz/Kg), $F_{m,n}$ (m=1)			
		n=1	n=2	n=3	n=4
Rings	Rectangular	34.873	15.775	15. 442	26.249
Stringers	Rectangular				
Rings	C-Shape	34.893	14.773	11.291	16.855
Stringers	C-Shape				
Rings	I-Shape	32.518	15.223	14.610	25.721
Stringers	I-Shape				
Rings	Ω -Shape	34.492	14.693	11.626	17.681
Stringers	Ω -Shape				
Rings	C-Shape	34.224	14.806	11.148	16.262
Stringers	I-Shape				
Rings	I-Shape	33.006	15.586	14.548	26.669
Stringers	C-Shape				
Rings	Rectangular	34.162	15.345	15.256	25.124
Stringers	I-Shape				
Rings	I-Shape	33.047	15.678	14.629	26.752
Stringers	Rectangular				

Table 7. Material properties of an isotropic cylindrical shell (non-rotating and without stiffeners).

Material type	Ti-6Al-4V	Aluminum	Si3N4	Zirconium	Stainless steel
density (Kg/m^3)	2370	2685	4429	5700	7800
Young modulus (GPa)	122.55	72.40	348.43	244.27	201.04
Poisson ratio	0.29	0.33	0.24	0.29	0.33

Table 8. Ratio of natural frequencies to weights (Kg) of an isotropic cylindrical shell (non-rotating and without stiffeners) for different types of materials, h=0.01 (m), R=1 (m), L=1 (m).

m	n	Ratio of natural frequencies to weights (Hz/Kg), $F_{m,n}$				
		Ti-6Al-4V	Aluminum	Si3N4	Zirconium	Stainless steel
1	1	158.966	100.915	105.468	60.171	33.962
1	2	86.259	54.803	57.187	32.651	18.444
1	3	73.296	46.493	48.678	27.743	15.647
1	4	147.320	93.898	97.286	55.763	31.601

tion is to maximize the defined parameter, $F_{m,n}$. As it can be seen in the first four rows of Table 6 (where ring and stringer shapes are the same), although for $m = n = 1$, the target function is maximum for C-shape and rectangular shape and is minimized for I-shape, but also for $m = 1$ and $n = 3$ (where basic frequency occurs), the target function is maximized for rectangular shape and I-shape and is minimized for C-shape. In other words, the results for I-shape and C-shape are vice versa but rectangular stiffeners have higher contents of the target function. Thus, rectangular shape is the best shape for stiffeners.

Also, combinations of shape types are considered to study the target function which results are in the second four rows of Table 6. The combination of C-shape and I shape for stiffeners, if the target function is the first natural frequency of structure, C-shape for rings and I-shape for stringers is an optimized state. And if the target function is the based natural frequency of structure, I-shape for rings and C-shape for stringers is an optimized state. The results of combination of I-shape and rectangular shape show that rectangular shape for rings and I-shape for stringer is an optimized state for both the first and the basic natural frequencies.

4.3 Optimization of material type

By using different types of materials including Ti-6Al-4V, Aluminum, Si3N4, Zirconium and stainless steel (which material properties are listed in Table 7) for an isotropic cylindrical shell (non-rotating and without stiffeners), the results for the ratio of the natural frequencies to weights ($F_{m,n}$) are listed in

Table 9. Material properties of a laminated cylindrical shell (rotating and with stiffeners).

Material type	Density (Kg/m^3)	E_{11} (Gpa)	E_{22} (Gpa)	G_{12} (Gpa)	ν_{12}
T300/5208	1600	181.0	10.3	7.2	0.28
Kevlar49/epoxy	1400	76.0	5.5	2.3	0.34
E-glass/epoxy	1800	36.0	8.3	4.1	0.36
Boron/epoxy	2000	204.0	18.5	5.6	0.23

Table 10. Ratio of natural frequencies to weights (Kg) of an isotropic cylindrical shell (rotating and with stiffeners) for different types of materials, h=0.01 (m), R=1 (m), L=1 (m).

m	n	Ratio of natural frequencies to weights (Hz/Kg), $F_{m,n}$			
		T300/5208	Kevlar49/epoxy	E-glass/epoxy	Boron/epoxy
1	1	148.309	103.756	86.764	97.005
1	2	116.917	95.946	59.446	95.183
1	3	99.071	73.046	49.639	69.329
1	4	187.087	158.967	100.023	163.200

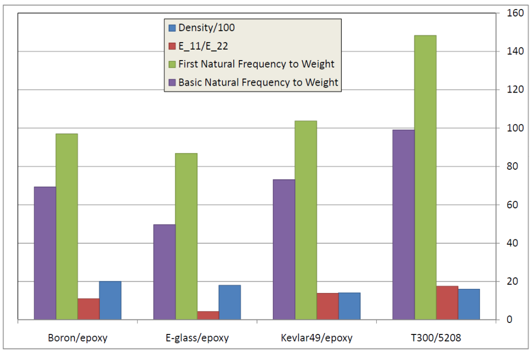

Fig. 3. Material type effect on natural frequencies, Young modulus and density of an isotropic cylindrical shell.

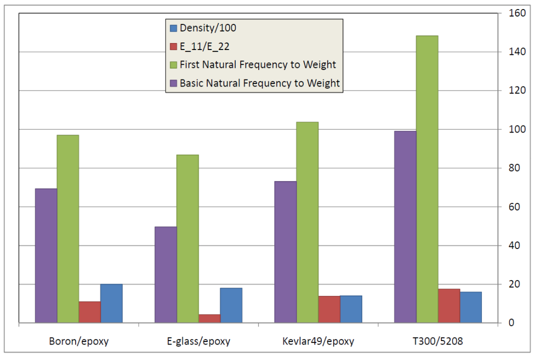

Fig. 4. Material type effect on natural frequencies, Young modulus and density of a laminated cylindrical shell.

Table 8.

The dimensions of the shell including thickness, radius and length are 0.01 (m), 1 (m) and 1 (m), respectively.

To understand the effect of density, Young modulus on the ratios of the first and basic natural frequency to weight, the contents are drawn in Fig. 3. In this figure, density and Young modulus are divided to 100 and 10, respectively, to have a better scale of comparison. Although higher density causes higher weight and lower natural frequencies and also higher Young modulus causes higher natural frequencies, but also the combination of density and Young modulus may have different effects on target function.

As it can be seen in Fig. 3 and Table 8, the maximum content of the target function is for Ti-6Al-4V material, which has the lowest weight in comparison with the others. Then, a second maximum content is for Si3N4 material, which has the highest content of Young modulus, although Aluminum material has a lower density and weight. That's due to the lower content of Young modulus of Aluminum material in comparison with Si3N4 material. The density of Aluminum is a half of the density of Si3N4 but the Young modulus of Si3N4 is approximately five times more than Aluminum.

The above procedure is repeated for a laminated cylindrical shell (rotating with stiffeners). Material types are considered as T300/5208, Kevlar49/epoxy, E-glass/epoxy and Boron/epoxy which material properties are listed in Table 9 and the results of the natural frequencies are in Table 10.

In this part, the angular velocity of shell is 0.1 (rev/s), and 10 rings (0.002 X 0.012) and 10 stringers (0.002 X 0.012) are considered with the same material properties of the shell. As it can be seen in Fig. 4, the first and the basic natural frequencies to weight ratio are maximized for T300/5208 material and minimized for E-glass/epoxy material due to high density and low Young modulus. Although the density of T300/5208 is more than Kevlar49/epoxy, but also higher content of Young modulus of T300/5208 causes to increase the stiffness and the natural frequencies of structure.

4.4 Optimization of isotropic shell

By using genetic algorithm, nine parameters are considered as the optimization variables. The parameters and changing limits are listed in Table 11. The other parameters such as material properties of the shell (non-rotating), rings and

Table 11. Defining of variables in optimization of isotropic rotating shell.

No.	Variables	Description	Dimension	Lower limit	Upper limit	Optimized contents
1	h	Thickness of shell	m	0.001	0.1	0.001
2	R	Radius of shell	m	0.05	1.5	0.05
3	L	Length of shell	m	0.5	2.5	0.5
4	N_r	Number of rings	-	0	20	1.707 (→ 2)
5	b_r	Width of rings	m	0.002	0.008	0.002
6	h_r	Height of rings	m	0.002	0.008	0.004
7	N_s	Number of stringers	-	0	20	0.057 (→ 1)
8	b_s	Width of stringers	m	0.002	0.008	0.002
9	h_s	Height of stringers	m	0.002	0.008	0.002

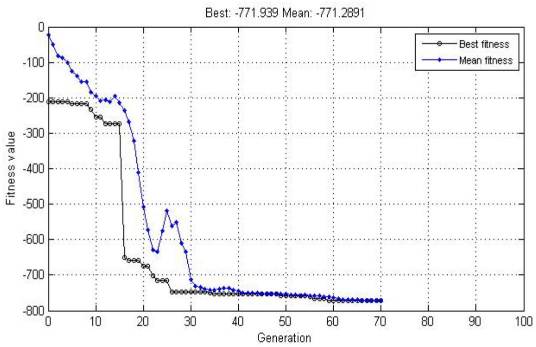

Fig. 5. Fitness value (best and mean) vs. generation for isotropic shell.

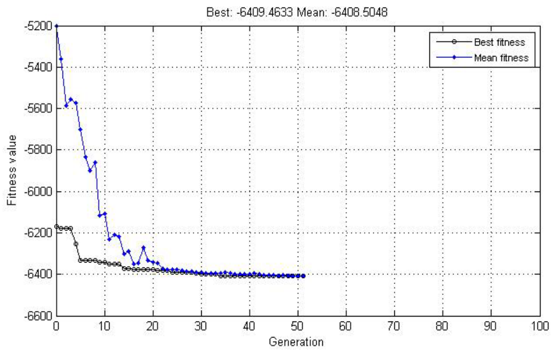

Fig. 6. Fitness value (best and mean) vs. generation for laminated shell.

stringers are constants including density defined as 7800 (Kg/m^3), Poisson ratio is defined as 0.33 and Young modulus is defined as 201.04 (GPa). For this case, nine parameters including h, R, L, the number of rings (N_r) and their dimensions (b_r, h_r), the number of strings (N_s) and their dimensions (b_s, h_s) are optimized. The defined target function should be maximized. By using optimization tool in MATLAB software and 70 generation (Fig. 5), the result is 0.001 (m), 0.05 (m) and 0.5 (m) for thickness, radius and length of the shell. The optimized contents of rings parameters are obtained as 1.707 (considering as 2), 0.002 (m) and 0.004 (m) and the optimized contents of stringers parameters are obtained as 0.057 (considering as 1), 0.002 (m) and 0.002 (m). By considering mentioned optimized contents of nine parameters, the weight of structure is become 3.500 (Kg) and the first natural frequency ($m = n = 1$) is 770.3 (Hz).

4.5 Optimization of laminated shell

In this part, a laminated cylindrical shell is considered with 23 variables for the optimization. The angular velocity and shell dimensions are constants. The variables include the numbers of rings and stringers and their dimensions and material properties, the numbers of layers and their angles and material properties of the shell. The changing intervals are listed in Table 12. By using optimization tool in MATLAB software and after 51 generations (Fig. 6), the optimized contents are obtained as shown in Table 12. Note that stiffeners have more effect on weight and low effect on the natural frequencies. It means that adding stiffeners causes a low change in the natural frequencies. At this situation, for a [0/60/60] laminated shell with 1 ring, the weight becomes 1.282 (Kg) and the first natural frequency (m = n = 1) becomes 992.3 (Hz) and the basic natural frequency (m = 1, n = 3) is 851.3 (Hz) which are higher than the characteristics of the isotropic shell.

5. Conclusions

Vibration analysis and genetic algorithm (GA) optimization of simply supported rotating cross-ply laminated stiffened cylindrical shell is performed under weight and frequency constraints. By using an energy approach, Rayleigh-Ritz method and Sander's relations, the governing equations are obtained. A good adoption is observed between the present results and other literatures in different type of results including isotropic shells, rotating laminated shells, stiffened isotropic shells and stiffened laminated shells. Then, stiffener shape and material properties are also optimized. As a result, both types of the below shells have better vibrational characteristics:

(1) An isotropic rotating cylindrical shell with rectangular-shape stiffeners made of Ti-6Al-4V material with 2 rings and 1 stringer.

(2) A [0/60/60] laminated rotating cylindrical shell with rec-

Table 12. Optimization parameters of laminated shell, h=0.001 (m), R=0.05 (m), L=0.5 (m) and Ω =0.1 (rev/s).

No.	Variables	Description	Dimension	Lower limit	Upper limit	Optimized values
1	N_r	Number of rings	-	0	10	0.06614 (→ 1)
2	b_r	Width of rings	m	0.002	0.012	0.00346
3	h_r	Height of rings	m	0.002	0.012	0.002
4	N_s	Number of stringers	-	0	10	0
5	b_s	Width of stringers	m	0.002	0.012	0.002
6	h_s	Height of stringers	m	0.002	0.012	0.00591
7	d	Density of shell	Kg/m^3	1400	2000	1400
8	d_r	Density of rings	Kg/m^3	1400	2000	1400
9	d_s	Density of stringers	Kg/m^3	1400	2000	1400
10	E_{11}	Young modulus of shell	GPa	35	205	67
11	E_{22}	Young modulus of shell	GPa	5	20	5
12	v_{12}	Poisson ratio of shell	-	0.2	0.4	0.4
13	G_{12}	Shear modulus of shell	GPa	2	8	6
14	E_r	Young modulus of rings	GPa	35	205	35
15	v_r	Poisson ratio of rings	-	0.2	0.4	0.21562
16	E_s	Young modulus of stringers	GPa	35	205	35
17	v_s	Poisson ratio of stringers	-	0.2	0.4	0.2
18	N_l	Number of layers	-	1	5	2.22745 (→ 3)
19	θ_1	Angle of first layer	degree	0	90	0
20	θ_2	Angle of second layer	degree	0	90	57.7266 (→ 60)
21	θ_3	Angle of third layer	degree	0	90	57.2958 (→ 60)
22	θ_4	Angle of forth layer	degree	0	90	33.6292
23	θ_5	Angle of fifth layer	degree	0	90	57.7266

tangular-shape stiffeners made of T300/5208 material with 1 ring.

Also, it should be mentioned the vibrational characteristics of an isotropic shell than a laminated shell. In other words, the natural frequencies of the laminated shell are higher than the isotropic one and the weight of the laminated shell is less than the isotropic one.

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