

# On the derivative constraints of input shaping control<sup>†</sup>

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## Abstract

Conventionally, derivative constraints have been added to the input shaper to increase robustness to modeling error in natural frequency and damping ratio, and the robustness of input shaping has been evaluated from the ratio of residual vibration amplitude with input shaping to that without input shaping. However, the derivative constraints used for the ZVD shaper and the derivative of the ratio of residual vibration amplitude are mathematically confused in the previous literatures, even if the conceptual explanation for both derivatives therein is generally acceptable. In this paper, the relationship of the derivative constraints used for ZVD shaper and the zero derivative of the ratio of residual vibration amplitude are derived and clarified mathematically, and the relationship between them is demonstrated using an example.

*Keywords:* Input shaping; Residual vibration; Derivative constraint; ZVD input shaper; Necessary and sufficient condition

## 1. Introduction

Input shaping control is an open-loop control technique among many other choices [1, 2] for reducing vibrations in flexible systems, which is implemented by convolving a sequence of impulses with a desired command. The amplitudes and time locations of the impulses are determined from the system's natural frequency and damping ratio by solving a set of constraint equations. Input Shaping<sup>®</sup> and Input Shaper<sup>™</sup> are trademarks of Convolve, Inc.

The early form of the input shaping control, called posicast control developed by O.J.M. Smith [3] in the late 1950's, was motivated by a simple wave cancellation concept for the elimination of the oscillatory motion of the under-damped system. However, it was sensitive to modeling errors of natural frequencies and damping ratios. Because of this sensitivity problem to parameter variations, and moreover the lack of microprocessor technology to implement this idea, his method has limited utility for real systems in the 1950s and 1960s, and consequently it did not come into widespread use.

The input shaping paper published in 1990 by Singer and Seering [4] renewed interest in pre-filtering reference inputs for residual vibration reduction, which improved robustness to modeling errors by adding additional constraints on the

derivative of residual vibration magnitudes. Given its robustness, input shaping control has been implemented using microprocessor technology on a variety of systems, including cranes [5, 6], disk drives [7], flexible spacecraft [8, 9], industrial robots [10, 11], and coordinate measuring machine [12]. Recently, a variety of input shaping has been introduced such as hybrid input shaping [13] and three-step input shaping techniques [14-16].

The robustness of the input shaping control has conventionally been evaluated using sensitivity curves plotted from the ratio of residual vibration amplitude with input shaping to that without input shaping at the time of the final impulse [7, 17-21]. However, the derivative constraints used for the ZVD (zero vibration and derivative) input shaper and the derivative of the ratio of residual vibration amplitude are mathematically confused in the previous literature, even if conceptual explanation for both derivatives therein is generally acceptable.

In this paper, the difference between the derivative constraints used for the ZVD or ZVDD input shaper, and the derivative of the ratio of residual vibration amplitude is clarified mathematically, and the relationship of the necessary and sufficient condition between the two derivatives is clarified. The validity of the discussion is demonstrated by example and sensitivity curves graphically.

Section 2 describes a derivation of derivative constraints for the ZVD input shaper, section 3 derives the derivative of the amplitude ratio of residual vibration, section 4 describes the relationship between two derivative constraints, section 5

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shows an example, and section 6 concludes the paper.

**2. Derivative constraints**

An input shaper suppresses residual vibration by generating an input that cancels its own vibration. The simplest one is the ZV (zero vibration) input shaper using two impulses, but the ZV input shaper is sensitive to modeling errors in actual undamped natural frequency  $\omega_n$  and damping ratio  $\zeta$ .

To increase robustness to modeling errors, the ZVD input shaper adds two constraints on derivatives

$$\frac{dC}{d\omega_n} = 0, \tag{1}$$

$$\frac{dS}{d\omega_n} = 0, \tag{2}$$

in addition to zero vibration constraints  $C = 0, S = 0$  when the actual natural frequency  $\omega_n$  equals to the modeled frequency  $\omega_m$  (fixed), where  $C$  and  $S$  are represented by

$$C(\omega_n) = \sum_{i=1}^3 A_i e^{\zeta\omega_n t_i} \cos(\omega_d t_i), \tag{3}$$

$$S(\omega_n) = \sum_{i=1}^3 A_i e^{\zeta\omega_n t_i} \sin(\omega_d t_i). \tag{4}$$

In (3) and (4),  $A_i, t_i$  are impulse amplitudes and impulse time that should be solved, and  $\omega_d$  represents damped natural frequency  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ . Eqs. (3) and (4) are easily derived from the time response of the second-order under-damped system with input  $A_1\delta(t) + A_2\delta(t - t_2) + A_3\delta(t - t_3)$ , and the condition that the response equals to zero at  $t \geq t_3$ .

Even if  $C$  and  $S$  are functions of  $\omega_n$  and  $\zeta$ , we fix the damping ratio  $\zeta$  for mathematical convenience in this paper, and thus  $C$  and  $S$  are considered as functions of only  $\omega_n$ .

Two constraints (1) and (2) are equivalent to the following two constraints, which is easy to show:

$$\sum_{i=1}^3 A_i t_i e^{\zeta\omega_n t_i} \cos(\omega_d t_i) = 0 \tag{5}$$

$$\sum_{i=1}^3 A_i t_i e^{\zeta\omega_n t_i} \sin(\omega_d t_i) = 0. \tag{6}$$

These two constraints are known as derivative constraints (or zero derivative constraints)[23, 24].

The best solution of  $A_i, t_i$  satisfying derivative constraints (5) and (6) with zero vibration constraints and  $\sum A_i = 1$  together, known as ZVD input shaper, is given by

$$t_1 = 0, \quad t_2 = \frac{\pi}{\omega_d}, \quad t_3 = \frac{2\pi}{\omega_d}$$

$$A_1 = \frac{1}{(1+K)^2}, \quad A_2 = \frac{2K}{(1+K)^2}, \quad A_3 = \frac{K^2}{(1+K)^2} \tag{7}$$

where

$$K = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}. \tag{8}$$

The process of adding robustness to modeling errors can be further extended to include the second derivatives and the third derivatives with respect to  $\omega_n$ . The general form of the  $p$ -th derivative constraints is given by

$$\sum_{i=1}^N A_i (t_i)^p e^{\zeta\omega_n t_i} \cos(\omega_d t_i) = 0 \tag{9}$$

$$\sum_{i=1}^N A_i (t_i)^p e^{\zeta\omega_n t_i} \sin(\omega_d t_i) = 0 \tag{10}$$

where  $N$  is the number of impulses of the input shaper.  $N$  must be increased for the existence of the solutions  $A_i, t_i$  as higher-order derivative constraints are added.

**3. Derivative of the vibration amplitude ratio**

When a sequence of  $N$  impulses (i.e., an input shaper)  $\sum A_i \delta(t - t_i)$  with  $\sum A_i = 1$ , is applied to the 2nd order under-damped system, the response  $y(t)$  can be easily obtained at  $t \geq t_N$  as follows:

$$y(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sqrt{C(\omega_n)^2 + S(\omega_n)^2} \sin(\omega_d t - \psi) \tag{11}$$

where  $\psi = \tan^{-1} S(\omega_n)/C(\omega_n)$ . The final index value 3 in (5) and (6) must be replaced with  $N$  for the  $C$  and  $S$  expressions in (11). Thus the ratio  $V$  of residual vibration amplitude with input shaping to that without input shaping can be represented by

$$V(\omega_n) = e^{-\zeta\omega_n t_N} \sqrt{C(\omega_n)^2 + S(\omega_n)^2}. \tag{12}$$

The expression in (12) is actually the ratio of the decay envelope magnitude of the input shaping case at  $t = t_N$  to the envelope magnitude of no input shaping case at  $t = 0$  as in the Ref. [30]. The ratio  $V$  in (12) can be considered as a robustness measure of the input shaper since sensitivity curves to modeling errors in  $\omega_n$  such as Fig. 1 may be drawn using (12).

In Fig. 1,  $\omega_n$  is the actual value of the undamped natural frequency of the system, and  $\omega_m$  is the modeled value of the undamped natural frequency  $\omega_n$ .

The derivative of  $V$  with respect to  $\omega_n$  is given by

$$\frac{dV}{d\omega_n} = -\zeta t_N e^{-\zeta\omega_n t_N} \sqrt{C(\omega_n)^2 + S(\omega_n)^2} \tag{13}$$

$$+ e^{-\zeta\omega_n t_N} \frac{C(\omega_n) dC/d\omega_n + S(\omega_n) dS/d\omega_n}{\sqrt{C(\omega_n)^2 + S(\omega_n)^2}}.$$

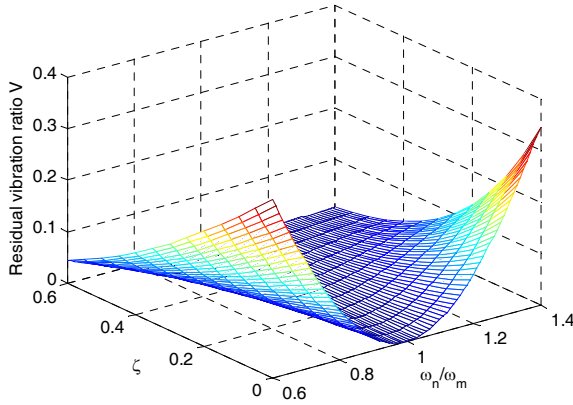


Fig. 1. Sensitivity curves to modeling errors in  $\omega_n$ .

However, obtaining the value of (13) at  $\omega_n = \omega_m$  under the zero vibration constraint is not so simple because  $C(\omega_n) = 0$  and  $S(\omega_n) = 0$  at  $\omega_n = \omega_m$ .

Let's evaluate the value of (13) by modifying the expression to

$$\frac{dV}{d\omega_n} = 0 + e^{-\zeta\omega_n t_N} \left\{ \frac{dC/d\omega_n}{\sqrt{1 + [S(\omega_n)/C(\omega_n)]^2}} + \frac{dS/d\omega_n}{\sqrt{1 + [C(\omega_n)/S(\omega_n)]^2}} \right\} \quad (14)$$

as  $\omega_n \rightarrow \omega_m$ . The zero term in (14) is due to zero vibration constraints. The value of

$$\lim_{\omega_n \rightarrow \omega_m} \frac{S(\omega_n)}{C(\omega_n)} \quad (15)$$

is needed to evaluate (14) as  $\omega_n \rightarrow \omega_m$ . The limit value of (15) is obtained from L'Hospital's rule such that

$$\lim_{\omega_n \rightarrow \omega_m} \frac{S(\omega_n)}{C(\omega_n)} = \lim_{\omega_n \rightarrow \omega_m} \frac{dS/d\omega_n}{dC/d\omega_n} = \lim_{\omega_n \rightarrow \omega_m} \frac{d^2S/d\omega_n^2}{d^2C/d\omega_n^2}. \quad (16)$$

In the second expression of (16), the limit value is still undetermined since the limit values of the numerator and the denominator all go to zero due to zero derivative constraints. However, the third expression of (16) has a limit value.

To conveniently obtain the second derivatives of  $C$  and  $S$  for ZVD series input shapers, we change the form of (3) and (4) into

$$C(\omega_n) = \sum_{i=1}^N A_i e^{(i-1)k \frac{\omega_n}{\omega_m}} \cos\left((i-1)\pi \frac{\omega_n}{\omega_m}\right) \quad (17)$$

$$S(\omega_n) = \sum_{i=1}^N A_i e^{(i-1)k \frac{\omega_n}{\omega_m}} \sin\left((i-1)\pi \frac{\omega_n}{\omega_m}\right) \quad (18)$$

where  $k = \zeta\pi/\sqrt{1-\zeta^2}$  is a constant. Let's obtain the second derivatives of (17) and (18) for a specific ZVD input

shaper with  $N = 3$ .

$$\begin{aligned} \frac{d^2C}{d\omega_n^2} &= A_2 (k/\omega_m)^2 e^{k \frac{\omega_n}{\omega_m}} \cos(\pi\omega_n/\omega_m) \\ &\quad - A_2 (k/\omega_m) e^{k \frac{\omega_n}{\omega_m}} (\pi/\omega_m) \sin(\pi\omega_n/\omega_m) \\ &\quad - A_2 (k/\omega_m) e^{k \frac{\omega_n}{\omega_m}} (\pi/\omega_m) \sin(\pi\omega_n/\omega_m) \\ &\quad - A_2 e^{k \frac{\omega_n}{\omega_m}} (\pi/\omega_m)^2 \cos(\pi\omega_n/\omega_m) \end{aligned} \quad (19)$$

$$\begin{aligned} &+ A_3 (2k/\omega_m)^2 e^{2k \frac{\omega_n}{\omega_m}} \cos(2\pi\omega_n/\omega_m) \\ &\quad - A_3 (2k/\omega_m) e^{2k \frac{\omega_n}{\omega_m}} (2\pi/\omega_m) \sin(2\pi\omega_n/\omega_m) \\ &\quad - A_3 (2k/\omega_m) e^{2k \frac{\omega_n}{\omega_m}} (2\pi/\omega_m) \sin(2\pi\omega_n/\omega_m) \\ &\quad - A_3 e^{2k \frac{\omega_n}{\omega_m}} (2\pi/\omega_m)^2 \cos(2\pi\omega_n/\omega_m), \end{aligned}$$

$$\begin{aligned} \frac{d^2S}{d\omega_n^2} &= A_2 (k/\omega_m)^2 e^{k \frac{\omega_n}{\omega_m}} \sin(\pi\omega_n/\omega_m) \\ &\quad + A_2 (k/\omega_m) e^{k \frac{\omega_n}{\omega_m}} (\pi/\omega_m) \cos(\pi\omega_n/\omega_m) \\ &\quad + A_2 (k/\omega_m) e^{k \frac{\omega_n}{\omega_m}} (\pi/\omega_m) \cos(\pi\omega_n/\omega_m) \\ &\quad - A_2 e^{k \frac{\omega_n}{\omega_m}} (\pi/\omega_m)^2 \sin(\pi\omega_n/\omega_m) \end{aligned} \quad (20)$$

$$\begin{aligned} &+ A_3 (2k/\omega_m)^2 e^{2k \frac{\omega_n}{\omega_m}} \sin(2\pi\omega_n/\omega_m) \\ &\quad + A_3 (2k/\omega_m) e^{2k \frac{\omega_n}{\omega_m}} (2\pi/\omega_m) \cos(2\pi\omega_n/\omega_m) \\ &\quad - A_3 (2k/\omega_m) e^{2k \frac{\omega_n}{\omega_m}} (2\pi/\omega_m) \cos(2\pi\omega_n/\omega_m) \\ &\quad - A_3 e^{2k \frac{\omega_n}{\omega_m}} (2\pi/\omega_m)^2 \sin(2\pi\omega_n/\omega_m). \end{aligned}$$

Therefore, the limit values of (19) and (20) are given by

$$\lim_{\omega_n \rightarrow \omega_m} \frac{d^2C}{d\omega_n^2} = \left( \frac{k^2 - \pi^2}{\omega_m^2} \right) \frac{2e^{2k}}{(1 + e^{2k})^2} \quad (21)$$

$$\lim_{\omega_n \rightarrow \omega_m} \frac{d^2S}{d\omega_n^2} = \left( \frac{2k\pi}{\omega_m^2} \right) \frac{2e^{2k}}{(1 + e^{2k})^2}. \quad (22)$$

Thus the limit value of (15) for the ZVD shaper is given by

$$\lim_{\omega_n \rightarrow \omega_m} \frac{S(\omega_n)}{C(\omega_n)} = \frac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1} \quad (23)$$

and the limit value of  $C(\omega_n)/S(\omega_n)$  for the ZVD shaper is given by the inverse of the value in (23). Substituting (23)

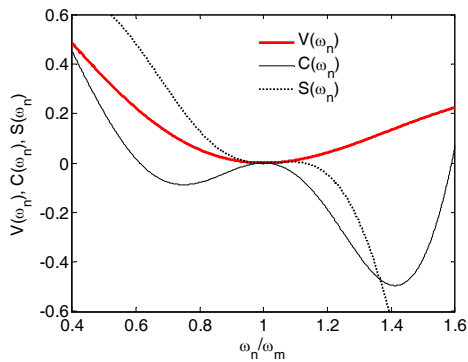


Fig. 2. An example demonstrating the fact (26), i.e.,  $dV/d\omega_n = 0$  iff  $dC/d\omega_n = 0, dS/d\omega_n = 0$ .

into (14), we can evaluate the derivative of  $V$  in (13) as  $\omega_n \rightarrow \omega_m$ .

This process of evaluating the derivative of  $V$  can be extended to the evaluation of the second derivative of  $V$ , and further to the evaluation of the third derivative of  $V$  at  $\omega_n = \omega_m$ .

**4. Relationship between two derivatives**

Some previous studies [7, 20, 24-32] describe that the ZVD input shaper is derived from zero vibration constraints and an additional constraint

$$\frac{dV}{d\omega_n} = 0 \tag{24}$$

at the modeling frequency  $\omega_n = \omega_m$ .

However, the zero derivative constraints for the ZVD shaper were originally the constraints in (1) and (2) instead of (24) as in the paper by Singer et al.[4]. The relationship of the two derivative constraints is not clearly published in the previous literatures.

It is clear that if constraints (1) and (2) are satisfied, then the constraint  $dV/d\omega_n = 0$  is satisfied by (14) and (23) as  $\omega_n \rightarrow \omega_m$  for the ZVD input shaper. To show the converse such that if  $dV/d\omega_n = 0$  is satisfied, then the constraint (1) and (2) is satisfied as  $\omega_n \rightarrow \omega_m$  for the ZVD input shaper, we evaluate the derivative (13) for any  $dC/d\omega_n$  and  $dS/d\omega_n$  using L'Hospital's rule as  $\omega_n \rightarrow \omega_m$ .

$$\begin{aligned} \frac{dV}{d\omega_n} &= e^{-\zeta\omega_n t_N} \frac{dC/d\omega_n + [S(\omega_n)/C(\omega_n)] dS/d\omega_n}{\sqrt{1 + [S(\omega_n)/C(\omega_n)]^2}} \\ &= e^{-\zeta\omega_n t_N} \sqrt{[dC(\omega_n)/d\omega_n]^2 + [dS(\omega_n)/d\omega_n]^2} \end{aligned} \tag{25}$$

Eq. (25) implies that the converse proposition is also true. Therefore, constraint (24) is a necessary and sufficient condition for constraints (1) and (2). That is, as  $\omega_n \rightarrow \omega_m$

$$\frac{dC}{d\omega_n} = 0, \frac{dS}{d\omega_n} = 0 \Leftrightarrow \frac{dV}{d\omega_n} = 0 \tag{26}$$

for the ZVD input shaper. This discussion can be extended to higher-order derivative constraints for ZVDD or ZVDDD input shapers. The fact of (26) implies that the satisfaction of (24) does guarantee the satisfaction of (1) and (2) as  $\omega_n \rightarrow \omega_m$  for ZVD series input shapers.

In the next section, one example demonstrating the fact (26) is given.

**5. An example**

Let's consider a vibratory system  $\omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$  with  $\zeta = 0.2, \omega_n = 2\pi$  (rad/s), and obtain an input shaper composed of three impulses satisfying the following constraints,

$$\begin{aligned} C(\omega_n) &= 0, \quad S(\omega_n) = 0 \\ \frac{dV}{d\omega_n} &= 0 \end{aligned} \tag{27}$$

at the modeling frequency with  $A_1 + A_2 + A_3 = 1$ . A numerical solution for these input shaper  $A_i, t_i$  satisfying (27) is given by

$$\begin{aligned} A_1 &= 0.4291, \quad A_2 = 0.4519, \quad A_3 = 0.1190 \\ t_1 &= 0, \quad t_2 = 0.5103, \quad t_3 = 1.0206 \end{aligned} \tag{28}$$

which is the same as the ZVD input shaper obtained from the derivative constraints (1) and (2).

Fig. 2 shows sensitivity curves of the input shaper (28) when the modeling frequency  $\omega_m$  is fixed, and actual natural frequency  $\omega_n$  is a variable. In Fig. 2, two black lines represent values of  $C(\omega_n)$  and  $S(\omega_n)$ , and the red thick solid line represents the value of  $V(\omega_n)$ . Conclusively, for the ZVD input shaper,  $dV/d\omega_n = 0$  if and only if (iff)  $dC/d\omega_n = 0$  and  $dS/d\omega_n = 0$  at the modeling frequency  $\omega_n = \omega_m$ .

**6. Conclusion**

Conventionally, derivative constraints are added to the input shaper to increase robustness to modeling error in natural frequency and damping ratio, and the robustness of input shaping is evaluated from the ratio  $V$  of residual vibration amplitude with input shaping to that without input shaping. However, the derivative constraints used for the ZVD or ZVDD shaper and the derivative of the ratio  $V$  of residual vibration amplitude are used confusingly in some previous literatures.

In this paper, two derivative constraints for ZVD series shapers are clarified mathematically: The derivative constraint  $dV/d\omega_n = 0$  is a necessary and sufficient condition for  $dC/d\omega_n = 0$  and  $dS/d\omega_n = 0$  as  $\omega_n \rightarrow \omega_m$ , and this result can be extended to higher-order derivative constraints for ZVD series shapers.

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