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Cam profile optimization for a new cam drive †

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Abstract

A complex cam shape optimization problem is studied to optimize a unique cam mechanism for a new cam drive engine. First, the optimization problem is defined through analyzing the unique cam mechanism. Multiple design specifications are included in the optimization problem by defining the output torque of the engine as the objective function and the contact stress, radius of curvature, and pressure angle as the constraints. Second, an analytical scenario is designed to find the best ever cam profiles through manipulating the different combinations of cam profile representations and optimization methods. Two types of curve representations, including general polynomial spline and B-spline, are employed in cam profile synthesis. In addition, both a classical optimization technique and a genetic algorithm (GA) based method are applied to solve the complex optimization problem. Finally, comparative studies are performed among the initial profile and the optimal profiles to demonstrate the effectiveness of these proposed design approaches on solving the cam profile optimization problem. Results show that the best profiles are obtained from a combination of the B-spline representation and the GA-based method. In addition, compared to the initial design, the engine performance is improved greatly by the proposed optimization approaches.

Keywords: Cam design; Cam drive engine; Shape optimization; Uniform B-spline

1. Introduction

As an innovation of replacing the conventional crankshaft/connecting rod mechanism by cam mechanism in engine design, a new type of engine called the cam drive engine is being developed in a few places in the world. The cam drive engine has unique features over the conventional engines, among which the cam drive is the most prominent one. For cam drive, the cam profiles control the engine operation strokes by determining the timing of various intake and exhaust events. Moreover, the use of cam drive facilitates having a separate compression ratio and expansion ratio, a separation that is much more difficult to accomplish with the prevalent crankshaft/connecting rod engine design. The new type of engine is believed to have major advantages over the conventional engines, including higher power, better fuel economy, smoother operation, higher reliability, and lighter weight.

The application of the cam mechanism to cam drive engine makes the design of the cam profiles essential to the overall engine performance. However, the initial cam profiles were designed using a traditional cam design approach. For the so-called trial-and-error approach, the cam profiles are first generated to meet the geometry specifications such as the specified values of displacement, velocity, and acceleration at different engine events, and then the feasibility of the generated cam profiles is checked to meet the other design specifications, including the output torque of the engine, radius of curvature, and pressure angle, etc.. Although this approach is simple in principle, its disadvantages are obvious. First, this ap-

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proach is not efficient. It has to be applied many times in order to achieve a satisfactory engine performance. Therefore, it results in a laborious and time-consuming design process. Moreover, since the cam design problem involves the determination of the profiles of both intake cam and exhaust cam, the interplays of the two profiles add more complexity to the trial-and-error approach. Finally, although the result found by this approach is feasible, it is very unlikely to be optimal. Therefore, it is necessary to use an optimal design approach, in conjunction with an appropriate cam profile representation, to determine the most suitable cam profiles of both intake cam and exhaust cam in order to meet the multiple design specifications. This is the motivation of the proposed research. To this end, we first review the previous research on cam design and optimization as follows.

The cam design has changed dramatically over the past decades by taking advantage of the tremendous advance in computing devices and mathematics tools, especially the splines. The impetus for the change is the demand for cams with higher speeds, smoother operation, and better performance. The research activities on cam design can be classified into two categories according to the curve representation of cam profile: polynomial-based methods and spline-based methods. For example, polynomial functions [1, 2] were used to design cam profiles, and the residual vibrations at a specific design speed were suppressed. These studies were extended by Chew [3] to minimize residual vibrations over a range of speeds about the design speed. In recent years, the trend of modern cam design is that splines are replacing polynomials as the mathematical representations of the cam profile because of their versatility, ease of application, and flexibility. For instance, B-splines [4, 5], and cubic splines [6] were applied to cam profile synthesis, respectively. Moreover, trigonometric splines [7] were used to generate a set of rational geometric splines with rational offsets, which is accepted by CNC machines.

The optimization methods used in cam design and optimization consist of both traditional methods and evolutionary methods. A number of traditional optimization methods have been applied, such as Lagrange multiplier technique [8], optimal control theory [9], and generalized reduced gradient methods [3], etc. For evolutionary methods, genetic algorithms (GAs) [10] were applied to cam profile optimization recently. A critical review on modern cam design can be found in [11].

In the review of the past studies on cam design and optimization, we address the following three aspects in this research. First, the multiple design specifications for the cam drive engine are considered in the optimization problem. We select the output torque at a specified engine speed as the objective function. In addition, the other important design specifications, including contact stress, radius of curvature, and pressure angle, are considered as the constraints in the optimization problem. The major difference of the proposed research from typical cam profile optimization is that the output torque is selected as the objective function instead of the residual vibrations. The reason is that the presence of residual vibrations is not prominent because the follower (See Fig. 1) is designed to have enough rigidity in the cam-follower mechanism. In contrast, the output torque at a specified engine speed is one of the major concerns from the engine design aspect. The choice of the output torque as the objective function provides multidisciplinary aspects to the proposed research, in which both cam design and engine design are included. Second, instead of studying only one cam in the past studies, the proposed research aims to optimize intake cam and exhaust cam simultaneously to make the cam-follower mechanism fulfill appropriately the different engine events during the whole engine cycle. The complexity of the optimization problem increases accordingly since both cams are designed in the optimization problem. Finally, to study comprehensively the optimization problem, an analytical scenario is designed to investigate how the optimization results are affected by different combinations of cam profile representations and optimization methods.

The rest of the paper is organized as follows: Section 2 defines the cam profile optimization problem through analyzing the unique cam mechanism in the cam drive engine. Section 3 presents the analytical scenario that aims to find the best ever cam profiles through manipulating the different combinations of cam profile representations and optimization methods. Finally, conclusions are drawn in Section 4.

2. Formulation of optimization problem

2.1 Cam drive engine

A 3D CAD model of the cam drive engine is illustrated in Fig. 1. The main moving parts of the engine consist of eight piston & follower-roller assemblies, two cams (intake cam and exhaust cam), and a main-shaft. There are four cylinders horizontally mounted around a circle, each 90 degrees apart. Two opposite pistons are mated to a single cylinder, forming a combustion chamber between the two piston heads. At the one end of each follower, there are two rollers contacting the cam surface. Therefore, this engine can be categorized as a type of axial, cylinder cam with form-closed, translating follower. As the pistons fire back and forth, the rollers in turn press on the surfaces of the cams and thus make the cams and the main-shaft rotate. Obviously, the engine works by converting the translation of the pistons into the rotation of the cams and the main-shaft.

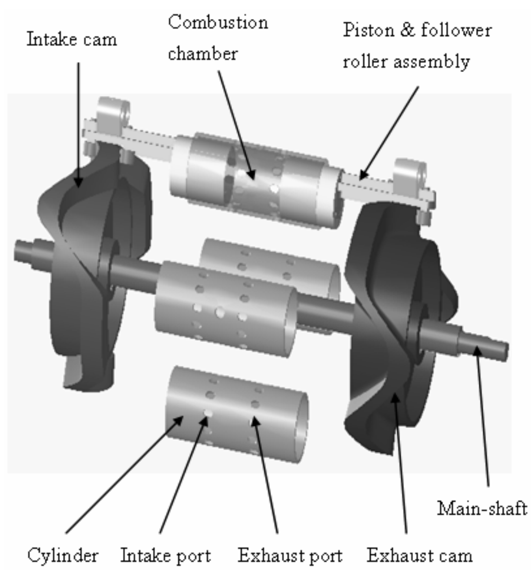

Fig. 1. 3D model of the 3-stroke engine.

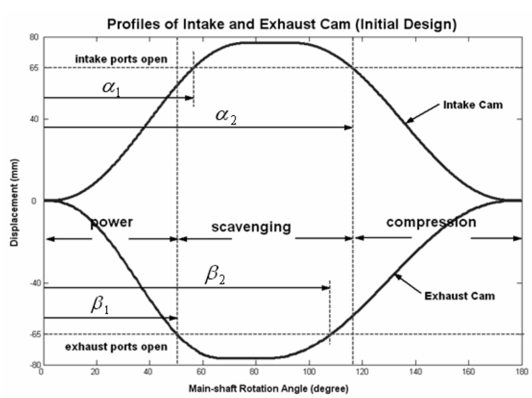

Fig. 2. Profiles of the cams (Initial design).

The engine function is fulfilled by coordinating the profiles of intake cam and exhaust cam. As shown in Fig. 2, there are three strokes over a period of 180 degrees: power, scavenging, and compression. The three strokes are determined by the rotation angles when intake (or exhaust) ports open (or close):

- $0-\beta_1$ (exhaust ports open at β_1): power stroke (intake ports close at α_2): scavenging stroke
- α_2-180° : compression stroke

where the angles α_1, α_2 and β_1, β_2 are determined to meet the design requirements of the engine. They are given as $\alpha_1 = 57^\circ$, $\alpha_2 = 115^\circ$ for intake cam, and $\beta_1 = 53^\circ$, $\beta_2 = 108^\circ$ for exhaust cam. The open or close phase is controlled by the movement of the pistons, which is in turn controlled by the shapes of the cam profiles. By properly designing the cam profiles, each cylinder has a 3-stroke cycle (including one power stroke) over a period of 180 degrees and hence one cylinder has four power strokes over two main-shaft revolutions (720 degrees), in contrast to one power stroke for the conventional engines.

2.2 Representation of cam profile

The design variables in the cam profile optimization problem are determined by parameterizing the cam profiles. In this study, two types of curve representations, including general polynomial spline and B-spline, are used to represent the cam profiles, respectively.

A. General Polynomial Spline

A four-piece general polynomial spline of order six is proposed to represent the cam profile. Thus, the profile of intake cam or exhaust cam has five knots, which are located at:

$t_1 = 0^\circ$, $t_2 = 10^\circ$, $t_3 = \alpha_1 = 57^\circ$, $t_4 = \alpha_2 = 115^\circ$, $t_5 = 180^\circ$ for intake cam

$t_1^{'} = 0^\circ$, $t_2^{'} = 10^\circ$, $t_3^{'} = \beta_1 = 53^\circ$, $t_4^{'} = \beta_2 = 108^\circ$, $t_5^{'} = 180^\circ$ for exhaust cam.

Each piece is represented by a 6-order polynomial in the form of

$$s(\theta) = a_5\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^5 + a_4\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^4 + a_3\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^3 + a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right)^2 + a_1\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) + a_0$$

$$\theta_1 \le \theta \le \theta_2 \tag{1}$$

where θ is the rotation angle of the main-shaft; θ_1 and θ_2 are the two end knots of the piece; $s(\theta)$ is the displacement of the cam profile (or the displacement of the piston); and $a_0, a_1, \cdots a_5$ are the coefficients of the polynomial function. These coefficients are defined as the design variables in the cam profile optimization problem. Therefore, for the representation of general polynomial spline, the number of design variables is calculated as: 6(order)×4(pieces)×2(cams)=48.

B. B-Spline

The B-spline curve is a piecewise polynomial curve defined by a set of control points which the curve ordinarily does not interpolate. A particular property of B-spline curve is local control, which refers to the ability to concentrate the effect of the change in curve shape within a specific region of the curve. It is also advantageous that the degree of the curve and the number of control points can be selected independently. These properties make B-spline a great candidate for geometry design and modeling. The B-spline curve is defined as [12]

Given integers $k \geq 1$, $m \geq 0$, and let

$$t_1 \leq t_2 \leq \cdots \leq t_k \leq t_{k+1} \leq \cdots \leq t_m \leq t_{m+1} \leq \cdots t_{m+k}$$

be a *knot sequence*. The B-spline curve $s(\theta)$, is written as

$$s(\theta) = \sum_{i=1}^{m} c_i B_{i,k}(\theta) \tag{2}$$

where $c_1, \cdots, c_m$ are the weighting parameters; $B_{i,k}(\theta)$ is the B-spline basis function for the k^{th} order polynomial representation. The B-spline curve is determined by selecting values for the weighting parameters and the knot sequence.

In this optimization problem, the cam profiles of both intake cam and exhaust cam are constructed by a quintic B-spline curve over the following knot sequence where the knots repeat at the ends

$$\begin{aligned} &0^\circ = t_1 = \cdots = t_6,\ \ t_7 = h,\ \ t_8 = 2h,\ \ \cdots, \\ &t_m = (m-6)h,\ \ t_{m+1} = \cdots = t_{m+6} = 180^\circ \end{aligned} \tag{3}$$

such that

$$s(\theta) = \sum_{i=1}^{m} c_i \hat{B}_{i,6}(\theta) \tag{4}$$

where $m \geq 6$, and $h = \frac{180^\circ}{m-5}$; $\hat{B}_{i,6} = B_{i,6}$ for $i = 6, 7, \cdots, m-5$, these are uniform B-spline basis functions. As for the other B-spline basis functions, $\hat{B}_{1,6}, \cdots, \hat{B}_{5,6}$ and $\hat{B}_{m-4,6}, \cdots, \hat{B}_{m,6}$, which have repeated knots and therefore are non-uniform, each of them can be expressed as a linear combination of uniform B-spline basis functions $B_{i,6}$ for $\theta \in [0^\circ,\ 180^\circ]$. Therefore, the cam profile defined in Eq. (4) can be considered as a uniform B-spline curve for computational purpose.

The knot sequence is fixed by selecting $m = 14$ and $h = 20^\circ$. Only the weighting parameters $c_1, \cdots, c_m$ are considered as the design variables in the optimization problem. Therefore, the number of design variables is calculated as: 2(cams) ×14=28.

2.3 Objective function

One of the important design specifications for conventional internal combustion engines is the maximum output (or mean) torque at a specified design speed. For the cam drive engine, the output torque is also the major concern, and thus it is selected as the objective function. The output torque of the engine is calculated as

$$\begin{aligned} \overline{T}(X) &= \frac{1}{180}\int_0^{180} T(X,\theta)d\theta \\ &= 4 \times \frac{1}{180}\int_0^{180}\left[T_{Intake}(X,\theta) + T_{Exhaust}(X,\theta)\right]d\theta \end{aligned} \tag{5}$$

where X is the vector of the design variables; $T_{Intake}(X,\theta)$ and $T_{Exhaust}(X,\theta)$ are the torques of the opposite pistons acting on intake cam and exhaust cam in one cylinder, respectively; the number "4" represents the multiplication for the four cylinders; 180-degree is a period of the engine. By applying force analysis to the cam mechanism, T_{Intake} is calculated as

$$\begin{aligned} T_{Intake}(X,\theta) = R_c\left[p(X,\theta)A_p - m_p a_{Intake}(X,\theta)\right] \\ \times \tan\phi_{Intake}(X,\theta) \end{aligned} \tag{6}$$

where R_c is the radius of the cam; A_p is the area of the cross section of the piston; m_p is the mass of the piston; $p(X,\theta)$ is the gas pressure within the cylinder, which can be predicted by a piecewise function of the cylinder volume (or the displacements of the pistons) according to different working phases of the engine. A detailed description of the gas pressure $p(X,\theta)$ is given in the Appendix; a_{Intake} is the acceleration of the piston; ϕ_{Intake} is the pressure angle of the profile of intake cam, and

$$\tan\phi_{Intake}(X,\theta)=\frac{v_{Intake}(X,\theta)}{R_c} \tag{7}$$

where $v_{Intake}(X,\theta)$ is the velocity of the piston. $T_{Exhaust}$ has a similar form

$$T_{Exhaust}(X,\theta)=R_c\left[p(X,\theta)A_p-m_p a_{Exhaust}(X,\theta)\right]\times\tan\phi_{Exhaust}(X,\theta) \tag{8}$$

The objective is to maximize the output torque expressed in Eq. (5):

$$\text{maximize } \bar{T}(X) \tag{9}$$

2.4 Constraints

A. Equality constraints

There are two types of equality constraints: boundary conditions for the representations of both general polynomial spline and B-spline, and smoothness equations only for the representation of general polynomial spline.

A1. Boundary conditions

The boundary conditions shown in Table 1 are taken into consideration. There are 20 boundary conditions in total.

Table 1. Boundary conditions.

Crucial points		Disp.	Vel.	Acc.
Intake cam	0°	0	0	0
	α_1	65 mm	---	---
	$\frac{\alpha_1+\alpha_2}{2}$	77 mm	0	---
	α_2	65 mm	---	---
	180°	0	0	0
Exhaust cam	0°	0	0	0
	β_1	65 mm	---	---
	$\frac{\beta_1+\beta_2}{2}$	77 mm	0	---
	β_2	65 mm	---	---
	180°	0	0	0

A2. Smoothness equations (Only for general polynomial spline)

For the smoothness equations, we consider the cam profiles continuous up to two times derivatives (or acceleration) at each internal knot t_2,t_3,t_4 and t_2',t_3',t_4' as defined in Section 2.2. For example, at knot t_2, the smoothness equation for velocity is

$$5a_5+4a_4+3a_3+2a_2+a_1=b_1 \tag{10}$$

where $a_1\sim a_5$ are the coefficients of the 6-order polynomial corresponding to the first piece ($0\sim t_2$) of intake cam; b_1 is the coefficient of the 6-order polynomial corresponding to the second piece ($t_2\sim t_3$) of intake cam. In total, there are 18 smoothness equations.

B. Inequality constraints

The radii of curvature of the cam profiles r should be checked over the whole engine cycle

$$\left|r_{Intake}(X,\theta)\right|\ge r_{roller},\quad 0^\circ\le\theta\le180^\circ \tag{11}$$

$$\left|r_{Exhaust}(X,\theta)\right|\ge r_{roller},\quad 0^\circ\le\theta\le180^\circ \tag{12}$$

where the radii of curvature $r_{Intake}(X,\theta)$ and $r_{Exhaust}(X,\theta)$ must be checked against the radius of the roller r_{roller} to avoid undercutting. In addition, over the engine cycle, both the maximum Hertzian contact stress between the cam and roller and the minimum pressure angle are also limited by the allowable values, respectively:

$$\sigma_{\max}(X)\le\sigma_a \tag{13}$$

$$\text{and}\quad \phi_{\min}(X)\ge\phi_a \tag{14}$$

where σ_a and ϕ_a are the allowable values of the Hertzian contact stress and the pressure angle.

In summary, the cam profile optimization problem is a complex optimization problem. The complexity arises from the interactions among the nonlinearity of the objective function, the large number of design variables (48 for the representation of general polynomial spline and 28 for the representation of B-spline), the large number of equality constraints (38 for the representation of general polynomial spline and 20 for the representation of B-spline), and the nonlinearity of inequality constraints. Therefore, it may cause difficulties for classical optimization methods in solving the complex problem.

3. Results and discussions

To study the complex cam profile optimization problem comprehensively, an analytical scenario is designed to investigate how the optimization results are affected by different combinations of cam profile representations and optimization methods. To this end, two types of curve representations, general polynomial spline and B-spline, are used in cam profile synthesis. Moreover, both a classical optimization technique and a genetic algorithm (GA) based method are applied to solve the optimization problem. Thereafter, a series of comparative studies are performed among the initial design and the optimal design of the cam profiles. The initial design of the cam profiles is set as the comparison reference, in which the cam profiles are designed by using a trial-and-error approach. The detailed analytical scenario is described as follows.

Investigation 1: General polynomial spline is used to represent the profiles of both intake cam and exhaust cam. Combined with the representation, a classical optimization method, named sequential quadratic programming (SQP), is applied to solve the constrained optimization problem. The main purposes of this investigation are to understand the characteristics of the optimization problem, and to find if the optimization results show any improvement over the initial design.

The profiles of the optimal design for both intake cam and exhaust cam are shown in Fig. 3 and Fig. 4. The initial design of the cam profiles is also included in the figures for comparisons. In addition, the torque curves of the optimal design and the initial design are illustrated in Fig. 5. To clearly demonstrate the results, a quantitative comparison of these profiles is made in Table 2. For the items compared in the table, the peak values of velocity and acceleration indicate the smoothness of the curve, or the smoothness of the piston movement. The higher the peak values, the worse the smoothness of the curve.

Although the mean torque value generated from the optimal design by using general polynomial spline increases by 18% compared to that for initial design, the smoothness of the optimal design becomes worse. The significant increase in the output torque is contributed mainly by the replacement of the trial-and-error approach by the optimization approach. On the other hand, the smoothness of the optimal profiles is deteriorated since only up to acceleration is maintained continuous over the internal knots of the optimal profiles. In contrast, maintaining good smoothness is the major concern when designing the initial cam profiles.

There are two ways to improve the smoothness while keeping the representation of general polynomial spline. One way is to add more smoothness equations to make sure that the higher order of derivatives are continuous at the internal knots. The other is to use more pieces to construct the profiles. However, both ways will consequently increase the complexity of the optimization problem due to the increase in the number of design variables and equality constraints (smoothness equations). Therefore, in order to improve the smoothness, and in the meantime not to increase the complexity of the optimization problem, the following investigation is designed.

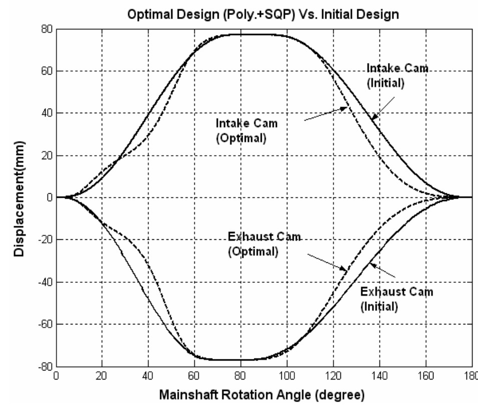

Fig. 3. Displacements of the initial and optimal profiles.

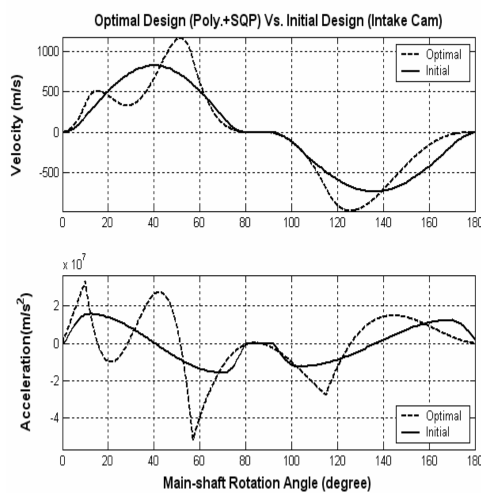

Fig. 4. Velocities and accelerations of the initial and optimal profiles.

Table 2. Comparison of four profiles.

	Velocity (m/s)			**Acceleration(m/s^2)**			**Output torque (Nm)**	**Improve-ment**
	Max.	Min.	Peak_to_Peak	Max.	Min.	Peak_to_Peak		
Initial design	14.4	-12.79	27.19	4811	-4811	9622	1140	Reference
Optimal (Poly.+SQP)	20.3	-16.95	37.25	10,130	-15,735	25,865	1345.1	**18% increase**
Optimal (B-spline+SQP)	12.6	-14.5	27.1	6,433	-4,307	10,740	1418.6	**24% increase**
Optimal (B-spline+GAs)	15.2	-11.3	26.5	7,447	-3,982	10,429	1457.6	**28% increase**

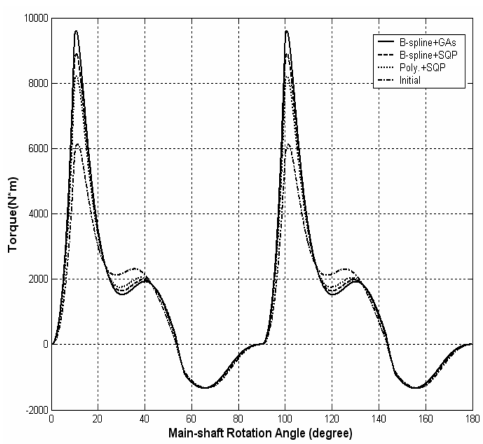

Fig. 5. Torques of the four design approaches.

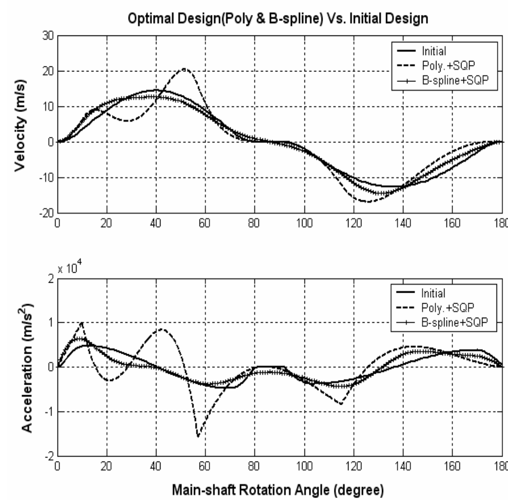

Fig. 7. Velocities and accelerations of the three profiles.

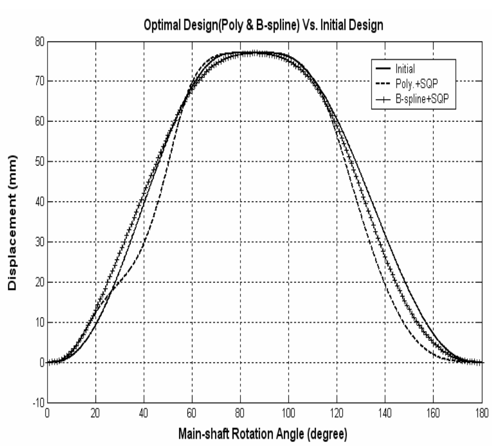

Fig. 6. Displacements of the three profiles.

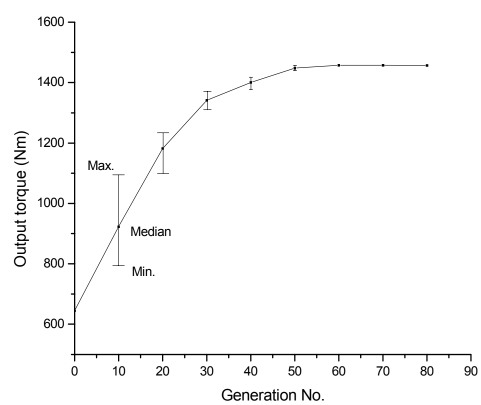

Fig. 8. Convergence history of the output torque.

Investigation 2: B-spline is used as the cam profile representation while the optimization method keeps the same. The purpose of this investigation is to examine if the B-spline representation is able to improve the smoothness. Both profiles of intake cam and exhaust cam are constructed over the knot sequence as defined in Eq. (3). The results are compared in Fig. 5 through Fig. 7, and Table 2. The following observations are made:

(1) The B-spline representation is much better than the representation of general polynomial spline. Compared to the optimal design using general polynomial spline, the optimal design using B-spline has great improvement on the maximum, minimum and peak_to_peak values of both the velocity and the acceleration. In the meantime, the output torque value generated by B-spline profiles is even higher than that generated by the profiles of general polynomial spline.

(2) Another advantage of the application of the B-spline representation is that the B-spline representation makes the optimization procedure much simpler than the representation of general polynomial spline. The number of design variables by using the B-spline representation (28) is much less than that for the representation of general polynomial spline (48). Moreover, for the B-spline representation, the number of equality constraints (20) is much less than that for the representation of general polynomial spline (38) because the smoothness equations are not needed.

(3) As shown in Table 2, the output torque provided by B-spline profiles increases by 24% (or 278Nm) compared to that provided by the initial profiles. At the same time, the smoothness of the B-spline profiles is comparable to that for the initial profiles.

(4) If comparing the higher order of derivatives of these profiles, such as jerk (order 3 or the derivative of acceleration), and ping (order 4), which indicates dynamic behaviors of the cam mechanism, the discontinuities and infinities occur over knots for the profiles using general polynomial spline since only up to acceleration continuity is maintained over the knots. However, for the profiles constructed by the quintic B-spline, the jerk and ping keep continuous over the whole range except for the beginning and end points.

Investigation 3: In the above two investigations, where the classical optimization technique SQP is used, it is observed in our experiments that the final solution is dependent on the initial guess of the solution. Therefore, it is very likely that the optimal solution found by SQP is a local optimal rather than a global optimal. To overcome the problem, an efficient constrained GA-based method [13] is selected to solve the optimization problem because GAs are capable of finding the global optimum. To investigate the performance of the GA-based method on solving the optimization problem, an experimental study is designed. The optimization procedure combined with both the GA-based method and the B-spline representation is run 12 times for every 10 generations from generation No. 10 to generation No. 80. All the runs start from different initial populations with the population size of 50. The convergence history of the output toque is illustrated in Fig. 8. For every 10 generations from generation No. 10 to generation No. 80, the maximum, minimum, and median values obtained from the 12 runs are plotted in the figure. The following observations are made:

(1) It can be observed in Fig. 8 that an acceptable convergence of the output toque is reached after generation No. 50. The maximum output torque with the value of 1457.6 Nm is found around generation No. 60. This value is slightly higher than the optimal output torque found in Investigation 2, where the classical method SQP is used. Moreover, it can be seen that there is no further improvement in the output torque after generation No. 60. Therefore, it is very likely that the optimal solution found by this investigation is a global optimum.

(2) The profile of intake cam generated from this investigation is compared to the other three profiles in Fig. 9 and Fig. 10. In addition, the smoothness values of these profiles are compared in Table 2. The results indicate that the smoothness values of the profile generated from the combination of the B-spline representation and the GA-based method are slightly better than that for the profile generated from the combination of the B-spline representation and SQP.

In summary, among the three sets of optimal profiles, the best profiles are obtained from a combination of the B-spline representation and the GA-based method through the designed analytical scenario. The best profiles provide better results in terms of both

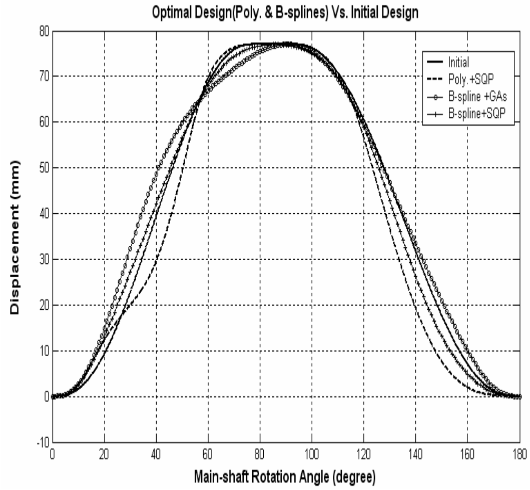

Fig. 9. Displacements of the four profiles.

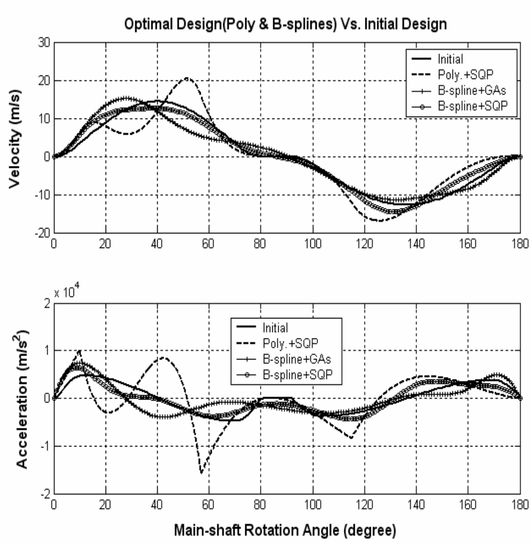

Fig. 10. Velocities and accelerations of the four profiles.

the output torque value and the smoothness values, compared to the profiles generated from the combination of the B-spline representation and SQP, and the combination the representation of general polynomial spline and SQP. In addition, the output torque generated by the best profiles increases significantly compared to that generated by the initial profiles. In the meantime, the smoothness of the best profiles is comparable to that for the initial profiles.

4. Conclusions

A complex cam profile optimization problem has been investigated for a unique cam mechanism in a new cam drive engine. First, the optimization problem has been defined by taking into account multiple design specifications. The output torque of the engine is considered as the objective function. In addition, the other design specifications, including the contact stress, the pressure angle, and the radius of curvature, are selected as the constraints. Second, an analytical scenario has been designed to investigate how the optimization results are affected by different combinations of cam profile representations and optimization methods. To this end, two types of curve representations, i.e., general polynomial spline and B-spline, have been used in cam profile synthesis. Moreover, both a classical optimization technique and a genetic algorithm (GA)-based method have been applied to solve the optimization problem. Finally, a series of comparative studies have been performed among the initial design and the optimal design of the cam profiles. Three sets of the optimal profiles have been generated through manipulating different combinations of the cam profile representations and the optimization methods. Results show that the best profiles are obtained from the combination of the B-spline representation and the GA-based method. The best profiles provide better results in terms of the output torque and the smoothness value, compared to the other two sets of optimal profiles. Moreover, the output torque generated by the best profiles increases by 28% compared to that generated by the initial profiles. At the same time, the smoothness of the best profiles is comparable to that for the initial profiles.

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Appendix

The gas pressure $p(X,\theta)$ within the cylinder can be predicted by thermodynamic principles, as given by the following piecewise formula:

For the process of rapid combustion, $p(X,180^{\circ})$ is the gas pressure at the end of compression; $p(X,\theta_{rc})$ is the gas pressure at the end of rapid combustion; and θ_{rc} indicates the period of the rapid combustion, which can be determined from engine experiments.

For the process of mixing-controlled combustion, $s_{in}(X,\theta)$ and $s_{ex}(X,\theta)$ are the displacement functions of intake cam and exhaust cam, respectively; Y_{tdc} is the distance of two opposite pistons when they reach at top dead center (TDC); and n_2 is the polytropic constant of the mixing-controlled combustion process, which can be empirically determined and further tuned by experiments.

For the process of scavenging, P_{In} is the intake pressure and can be determined from experiments.

For the process of compression, n_1 is the polytropic constant of the compression process, which can be empirically determined and further tuned by experiments.

Comparisons with the measured gas pressure from experiments show that the above equation is able to predict the gas pressure with an acceptable accuracy.

$$p(X,\theta)=\begin{cases} p(X,180^{\circ})+\dfrac{p(X,\theta_{rc})-p(X,180^{\circ})}{\theta_{rc}}\times\theta, & 0^{\circ}\le\theta<\theta_{rc} \quad \textit{Rapid Combustion} \\ p(X,\theta_{rc})\times\left(\dfrac{s_{in}(X,\theta_{rc})+s_{ex}(X,\theta_{rc})+Y_{tdc}}{s_{in}(X,\theta)+s_{ex}(X,\theta)+Y_{tdc}}\right)^{n_2}, & \theta_{rc}\le\theta\le\beta_1 \quad \textit{Mixing-controlled Combustion} \\ P_{In}+\dfrac{p(X,\beta_1)-P_{In}}{\alpha_2-\beta_1}\times(\alpha_2-\theta), & \beta_1<\theta\le\alpha_2 \quad \textit{Scavenging} \\ P_{In}\times\left(\dfrac{s_{in}(X,\alpha_2)+s_{ex}(X,\alpha_2)+Y_{tdc}}{s_{in}(X,\theta)+s_{ex}(X,\theta)+Y_{tdc}}\right)^{n_1}, & \alpha_2<\theta\le180^{\circ}, \quad \textit{Compression} \end{cases}$$

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